The impact of parameterized source positions on the CPO & free core nutation

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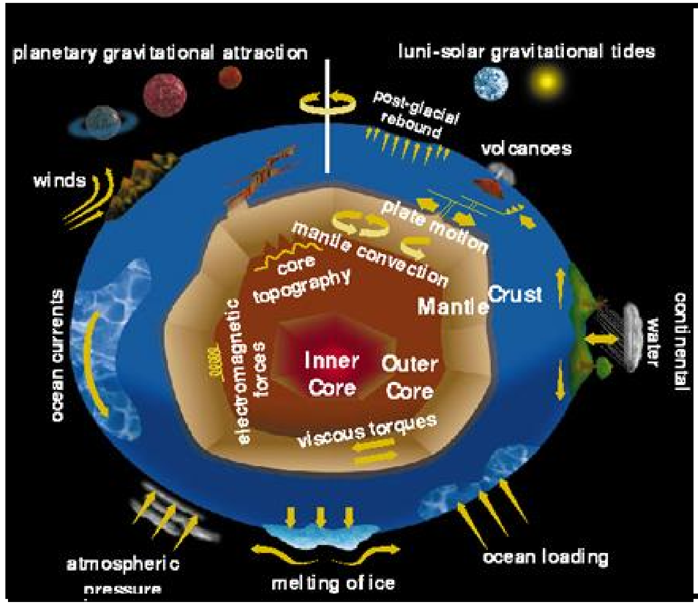
Contents

Earth Orientation Parameters (EOP)

Pole coordinates (x_p, y_p) **ERA ~ *dUT1*** **Celestial Pole Offsets (*dX* , *dY*)**

Forces that perturb the Earth's rotation



- Ice melting
- Redistribution of continental water
- Atmospheric transport and oceanic circulation
- Postglacial Rebound
- Winds
- Luni-solar tides
- Tectonic movements
- Free core nutation (FCN)
- …

VLBI is the only technique capable to provide and model the five EOP

IAU 2006/2000A precession-nutation model

The IAU 2006/2000A precession-nutation model is provided as a **series of luni-solar and planetary nutations in longitude and obliquity**, referred to the ecliptic of date, expressed as celestial pole coordinates X and Y with their time variations.

See Chapter 5 - IERS Conventions (2010) (Petit & Luzum 2010)

$$\text{ARGUMENT} = \sum_{j=1}^{5} N_j F_j,$$

$$\begin{aligned} X = {} & -0.016617'' + 2004.191898''t - 0.4297829''t^2 \\ & -0.19861834''t^3 + 0.000007578''t^4 + 0.0000059285''t^5 \\ & + \textstyle\sum_i [(a_{s,0})_i \sin(\text{ARGUMENT}) + (a_{c,0})_i \cos(\text{ARGUMENT})] \\ & + \textstyle\sum_i [(a_{s,1})_i t \sin(\text{ARGUMENT}) + (a_{c,1})_i t \cos(\text{ARGUMENT})] \\ & + \textstyle\sum_i [(a_{s,2})_i t^2 \sin(\text{ARGUMENT}) + (a_{c,2})_i t^2 \cos(\text{ARGUMENT})] \\ & + \cdots , \end{aligned}$$

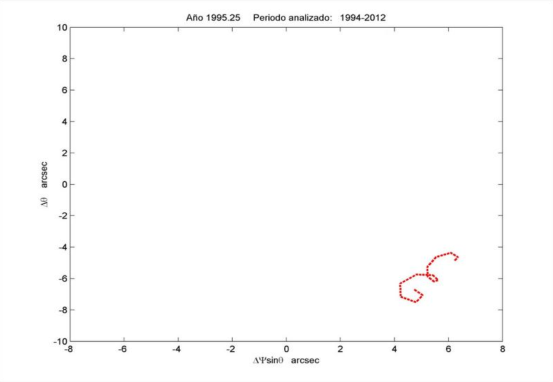
$$\begin{aligned} Y = {} & -0.006951'' \ldots \\ & +0.00190059 \ldots \\ & + \textstyle\sum_i [(b_{c,0})_i \ldots \\ & + \textstyle\sum_i [(b_{c,1})_i t \ldots \\ & + \textstyle\sum_i [(b_{c,2})_i t \ldots \\ & + \cdots , \end{aligned}$$

$$\begin{aligned} F_1 \equiv l & = \textit{Mean Anomaly of the Moon} \\ & = 134.96340251^\circ + 1717915923.2178''t + 31.8792''t^2 \\ & \quad +0.051635''t^3 - 0.00024470''t^4, \\ F_2 \equiv l' & = \textit{Mean Anomaly of the Sun} \\ & = 357.52910918^\circ + 129596581.0481''t - 0.5532''t^2 \\ & \quad +0.000136''t^3 - 0.00001149''t^4, \end{aligned}$$

… 512″t^2 … Sun … 706″t^2 … of the Moon … ″t^2

Table 5.2a: Extract from Table 5.2a (available at < [2] >) for the largest non-polynomial terms (*i.e.* coefficients of the Fourier and Poisson terms, with t in Julian centuries) in the development (5.16) for $X(t)$ compatible with the IAU 2006/2000A precession-nutation model (unit μas). The expressions for the fundamental arguments appearing in columns 4 to 17 are given by Eq. (5.43) and Eq. (5.44). (Because the largest terms are all luni-solar, columns 9-17 contain only zeros in the extract shown.)

i	$(a_{s,0})_i$	$(a_{c,0})_i$	l	l'	F	D	Ω	L_{Me}	L_{Ve}	L_E	L_{Ma}	L_J	L_{Sa}	L_U	L_{Ne}	p_A
1	−6844318.44	1328.67	0	0	0	0	1	0	0	0	0	0	0	0	0	0
2	−523908.04	−544.75	0	0	2	−2	2	0	0	0	0	0	0	0	0	0
3	−90552.22	111.23	0	0	2	0	2	0	0	0	0	0	0	0	0	0
4	82168.76	−27.64	0	0	0	0	2	0	0	0	0	0	0	0	0	0
5	58707.02	470.05	0	1	0	0	0	0	0	0	0	0	0	0	0	0
.....																
i	$(a_{s,1})_i$	$(a_{c,1})_i$	l	l'	F	D	Ω	L_{Me}	L_{Ve}	L_E	L_{Ma}	L_J	L_{Sa}	L_U	L_{Ne}	p_A
1307	−3309.73	205833.11	0	0	0	0	1	0	0	0	0	0	0	0	0	0
1308	198.97	12814.01	0	0	2	−2	2	0	0	0	0	0	0	0	0	0
1309	41.44	2187.91	0	0	2	0	2	0	0	0	0	0	0	0	0	0
.....																



Introduction: CPO & Free Core Nutation (FCN)

- Consideration of the Free Core Nutation (FCN) signal is necessary to improve the modelling of the Celestial Pole Offsets (CPO), since it is the major source of inaccuracy or unexplained time variability with respect to IAU2000 nutation theory.

- FCN can be excited by different geophysical processes not fully understood yet relative to the inertial frame (GCRS).

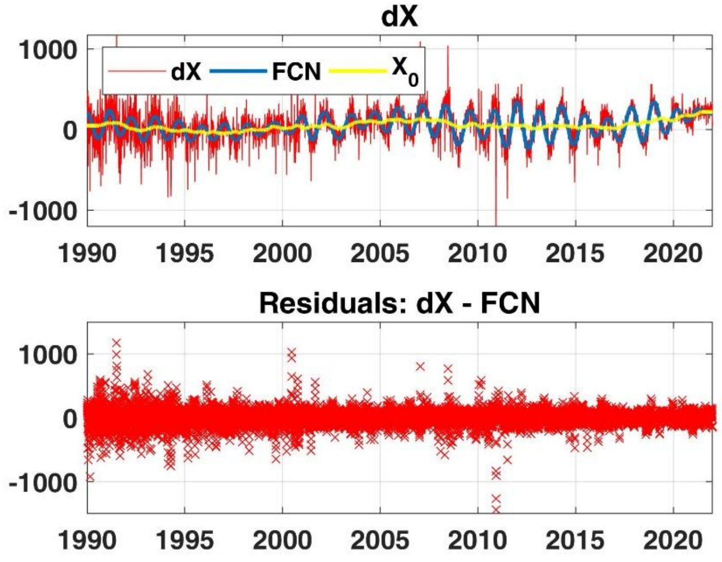
- VLBI is the only technique capable of accurately determining this signal.

- It has a long retrograde period of about 430 mean solar days (with average amplitude of about 100 µas) relative to the inertial frame.

- Nowadays, different empirical FCN models, derived by procedures with various levels of complexity are available.

- The accurate estimation of the FCN period is a challenging prospect. But, is there any evidence that the period of the FCN varies with time? In 2000 it was unknown whether or not it did. If so, then this would complicate making a model of it.

FCN model

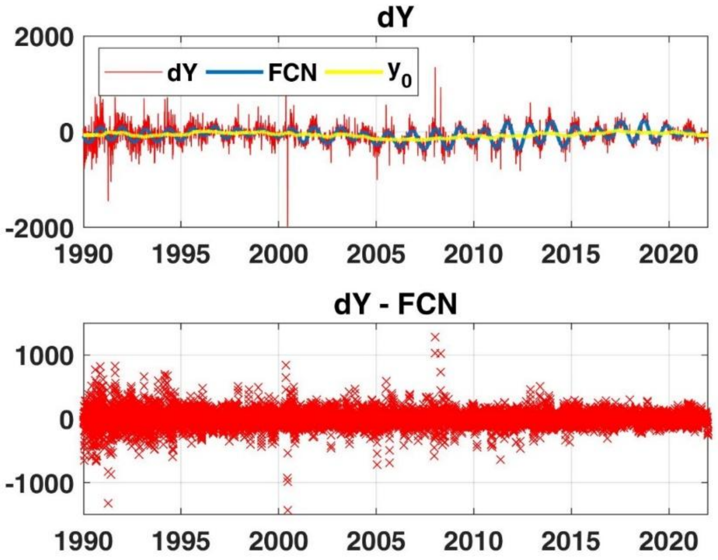


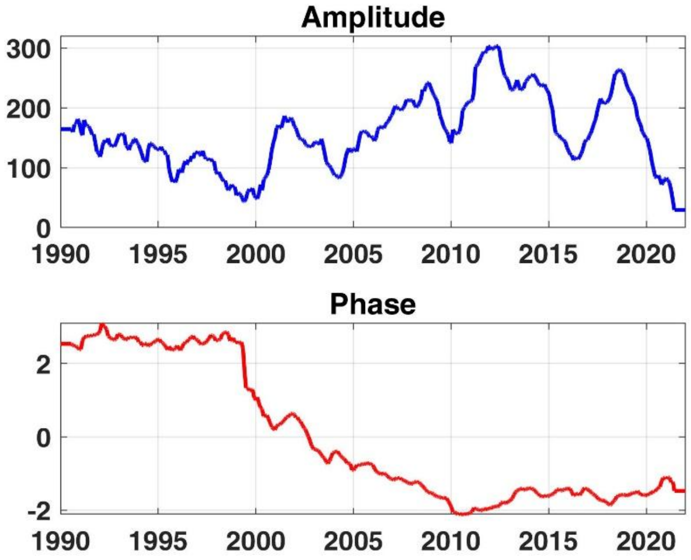
FCN models can be approached from computing a weighted least squares fit of these equations:

$$X_{FCN} = A_C \cos(\sigma_{FCN} t) - A_S \sin(\sigma_{FCN} t) + X_0$$

$$Y_{FCN} = A_S \cos(\sigma_{FCN} t) + A_C \sin(\sigma_{FCN} t) + Y_0$$

Amplitude coefficients are typically estimated by using a sliding window approach with a specific width (e.g 400 days, see Belda et. al 2016).





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- Introduction
- Inconsistencies/Problems
- The reassessment of the precession and nutation terms in analogy to the IAU 2006/2000A

Inconsistencies: Problem 1

ITRF

- VLBI
- SLR
- GNSS
- DORIS

ITRF and ICRF are realized separately by applying two individual computation methods.

ICRF

- VLBI

Not completely consistent

- **Reference Frames**
- **Earth Orientation Parameters (EOP)**

[1]GGOS goals for CRF and TRF: 30 µas and 3 µas/yr

(1mm position and 0.1 mm/year)

[2]JWG ITMER & CRTCE goals for EOP: 30 µas

ICRF3 accuracy of orientation is ~ 10 µas *(Ma et al. 2009).*

[1]GGOS: Global Geodetic Observation System

[2] WG Improving Theories and Models of the Earth's Rotation (ITMER)
WG Consistent Realization of TRF, CRF and EOP (CRTCE)

Inconsistencies: Problems 2

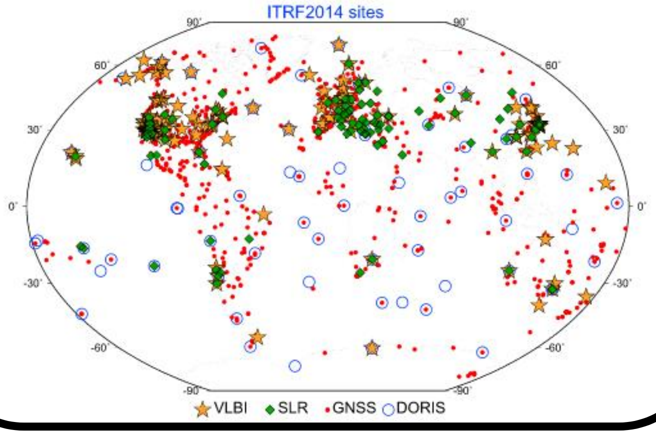
Why cannot we achieve the EOP with better accuracy?

- Poor geographical distribution of observing sites
- Insufficient number of radio telescopes (VLBI)
- Systematic errors of the space geodetic techniques
- Different Time domain of frames
  - ICRF3 <2015
  - ICRF2 <2009
  - ITRF2014 <2014
  - ITRF2008 <2008
- Different models applied

  ICRF2/ICRF3: atmospheric loading model

  ITRF2014: no non-tidal atmospheric loading model
- Incompleteness of the theory/models. (IAU 2006/2000A)
- …

Altamimi et al. (2016)



Inconsistencies: Problems 3

For geodesy, the radio sources are the most stable remote targets.
ICRF3 *is the most precise and stable frame available.*

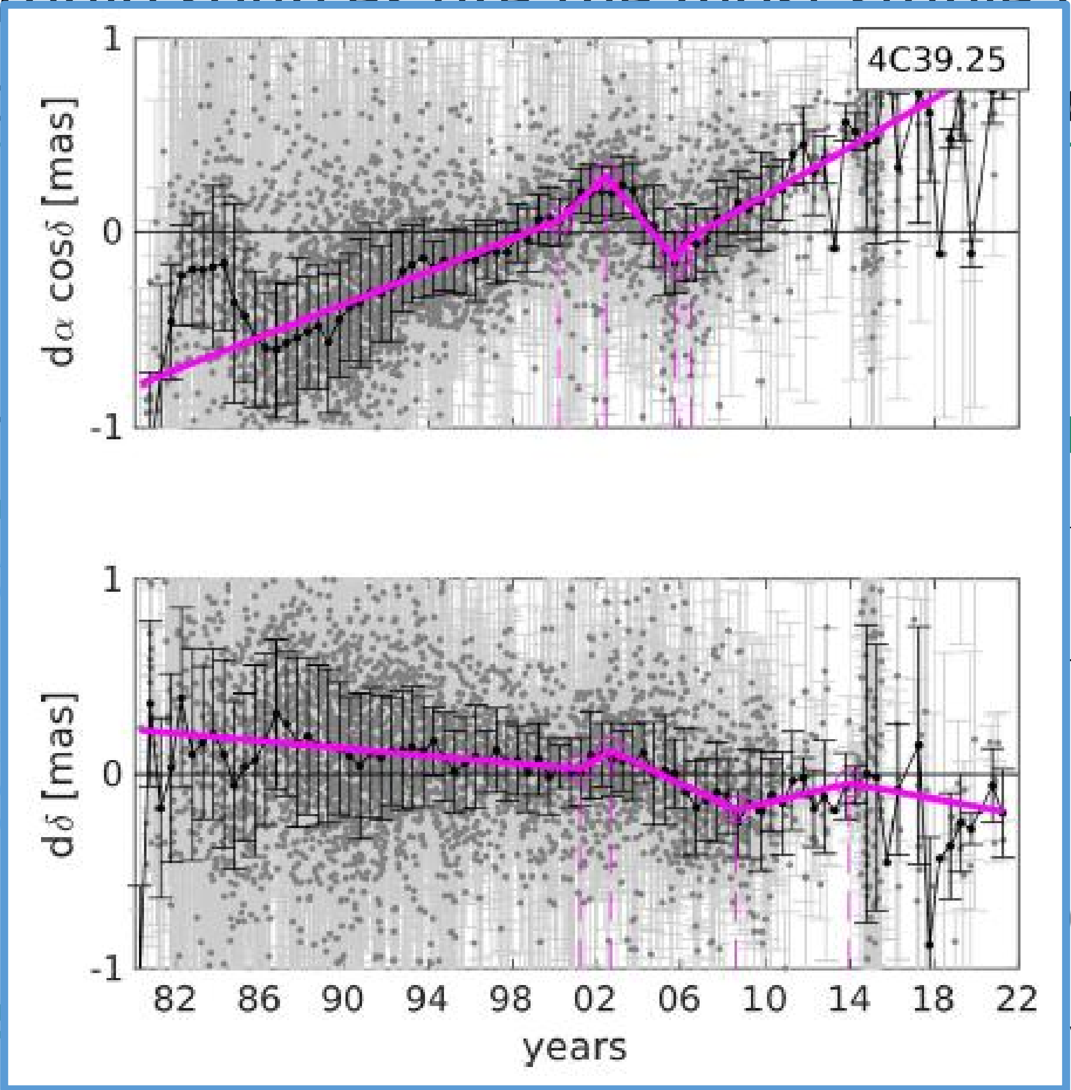
BUT…

geodetic VLBI considers radio source positions as **time-invariant**, i.e. they have **no apparent proper motion**.

ICRF Designation (1)	IERS Des. (2)	Inf. (3)	Right Ascension J2000.0 h m s	Declination J2000.0 o ' "	Uncertainty R.A. s	Uncertainty Dec. "	Corr. RA-Dc	Mean MJD	First MJD	Last MJD	Nb sess.	Nb del.	Nb rat.
ICRF J000020.3-322101	2357-326		00 00 20.39997606	-32 21 01.2337415	0.00000804	0.0002624	-0.0602	56559.8	52306.7	57776.0	4	237	0
ICRF J000027.0+030715	2357+028		00 00 27.02251377	+03 07 15.6463606	0.00005931	0.0003421	-0.0119	57974.7	57974.7	57974.7	1	28	0
ICRF J000053.0+405401	2358+406		00 00 53.08106320	+40 54 01.8096518	0.00001504	0.0002670	-0.1654	56460.2	50242.8	57809.9	3	152	0
ICRF J000105.3-155107	2358-161		00 01 05.32873479	-15 51 07.0752302	0.00000702	0.0002261	-0.2106	56338.4	50632.3	58137.6	4	316	0
ICRF J000107.0+605122	2358+605		00 01 07.09981547	+60 51 22.7980875	0.00003378	0.0001948	0.1619	57160.2	52306.7	57836.8	3	172	0

Note that according to **Karbon et. Al 2017**, **Multi-adaptive regression splines algorithm** (MARS) mitigates source position variations and thus allows the inclusion of 'unstable' sources into the datum definition.

For geodesy, the radio sources are the most stable remote targets.
ICRF3 *is the most* … *e.*



BUT...

geodetic VLBI co… **me-invariant**, i.e. they have **no app**…

ICRF Designation (1)	IERS Des. (2)	Inf. (3)	Right A… J2000.0… h m s…	Mean MJD	First MJD	Last MJD	Nb sess.	Nb del.	Nb rat.
ICRF J000020.3-322101	2357-326		00 00 2…	56559.8	52306.7	57776.0	4	237	0
ICRF J000027.0+030715	2357+028		00 00 2…	57974.7	57974.7	57974.7	1	28	0
ICRF J000053.0+405401	2358+406		00 00 5…	56460.2	50242.8	57809.9	3	152	0
ICRF J000105.3-155107	2358-161		00 01 0…	56338.4	50632.3	58137.6	4	316	0
ICRF J000107.0+605122	2358+605		00 01 0…	57160.2	52306.7	57836.8	3	172	0

Note that acco… …17, **Multi-adaptive regression splines** … s source position variations and thus allows the inclusion of 'unstable' sources into the datum definition.

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- **Objective and Workflow**

Goals / Objectives

Goal 1: To empirically evaluate the consistency, systematics and deviations of the IAU 2006/2000A precession-nutation model[1] using VLBI-based celestial pole offsets and MARS for source parametrization.

Goal 2: What is the impact on the CPO/FCN models

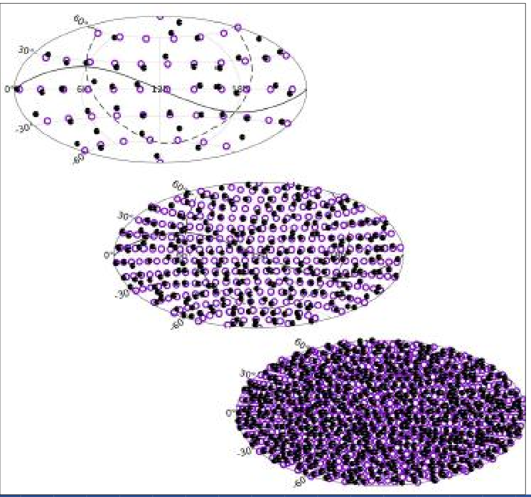
[1]IAU2000A was fitted to observations in 2000 using only 21 terms (Herring et al. 2002)

Workflow: VLBI analysis and a priori EOP

Standard VLBI analysis

Source parametrization

Different number of defining sources (N=100, 400, 800)



Session-wise

Global Solution

N=number of defining sources

A priori CPO:

ERP: IERS 14 C04

$$X_{VLBI} = (X_{IERS/USNO})_{apriori} + dX$$
$$Y_{VLBI} = (Y_{IERS/USNO})_{apriori} + dY.$$

IAU2006/2000A

A priori CPO:

$$X_{VLBI} = (X_{IAU2006/2000A})_{apriori} + dX$$
$$Y_{VLBI} = (Y_{IAU2006/2000A})_{apriori} + dY;$$

New Celestial Pole Coordinates

Reassessment of the precession and nutation (IAU 2006/2000A)

Estimation of new Empirical FCN models (Amplitude and phase)

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Corrections to the main nutation amplitudes

		Median Amplitude		Range		Median error	
Period (days)	CPO	As	Ac	As	Ac	As	Ac
-6.798.383	dX	20.609	40.782	23.125	13.122	4.271	1.692
	dY	15.177	-55.689	12.357	13.390	4.194	2.892
-3.399.192	dX	-2.489	13.002	5.952	10.743	3.790	1.756
	dY	-12.040	-19.373	4.917	18.238	3.696	2.902
-1.615.748	dX	3.043	-1.345	6.451	10.819	3.840	1.723
	dY	-1.051	-13.605	6.156	5.061	3.665	2.860
182.621	dX	3.848	-11.758	4.740	7.732	3.858	1.680
	dY	22.944	15.171	6.204	9.142	3.679	2.739
169.002	dX	11.513	3.608	7.009	11.740	3.799	1.668
	dY	1.960	3.753	7.563	6.114	3.650	2.745
91.313	dX	0.502	-6.366	5.969	3.272	3.704	1.639
	dY	-5.571	0.837	6.877	9.613	3.561	2.689
29.531	dX	0.952	3.380	7.026	4.027	3.877	1.683
	dY	8.665	8.457	6.182	3.131	3.714	2.771
27.555	dX	-6.677	-14.851	8.718	4.219	3.910	1.687
	dY	15.665	-7.885	2.174	5.990	3.739	2.773
27.333	dX	4.098	-3.220	5.442	5.760	3.821	1.668
	dY	5.379	-11.341	5.776	11.531	3.670	2.731
13.661	dX	-23.080	-8.482	14.510	3.865	3.835	1.657
	dY	3.894	1.631	9.298	4.347	3.672	2.715
13.579	dX	8.690	5.070	8.917	11.912	3.806	1.661
	dY	1.330	-1.867	4.297	8.285	3.641	2.719
8.910	dX	0.472	-5.762	5.528	1.771	4.065	1.678
Units: µas	dY	-9.100	-4.188	12.334	7.785	3.904	2.846

Table contains a summary of the most **significant amplitude deviations**, ordered by decreasing absolute values of the period.

Only the terms with amplitude corrections larger than **3 times the median error** are displayed.

Statistical analysis of the residuals

- The total number of frequencies used in the fit was 116
- We found 12 frequencies with amplitude corrections larger than **3 times the median error (≈3 μas)**
- Next table contains a summary of the **residuals using or not using MARS in the VLBI analysis** after fitting the main nutation amplitudes of the IAU 2006/2000A precession-nutation models and FCN model:

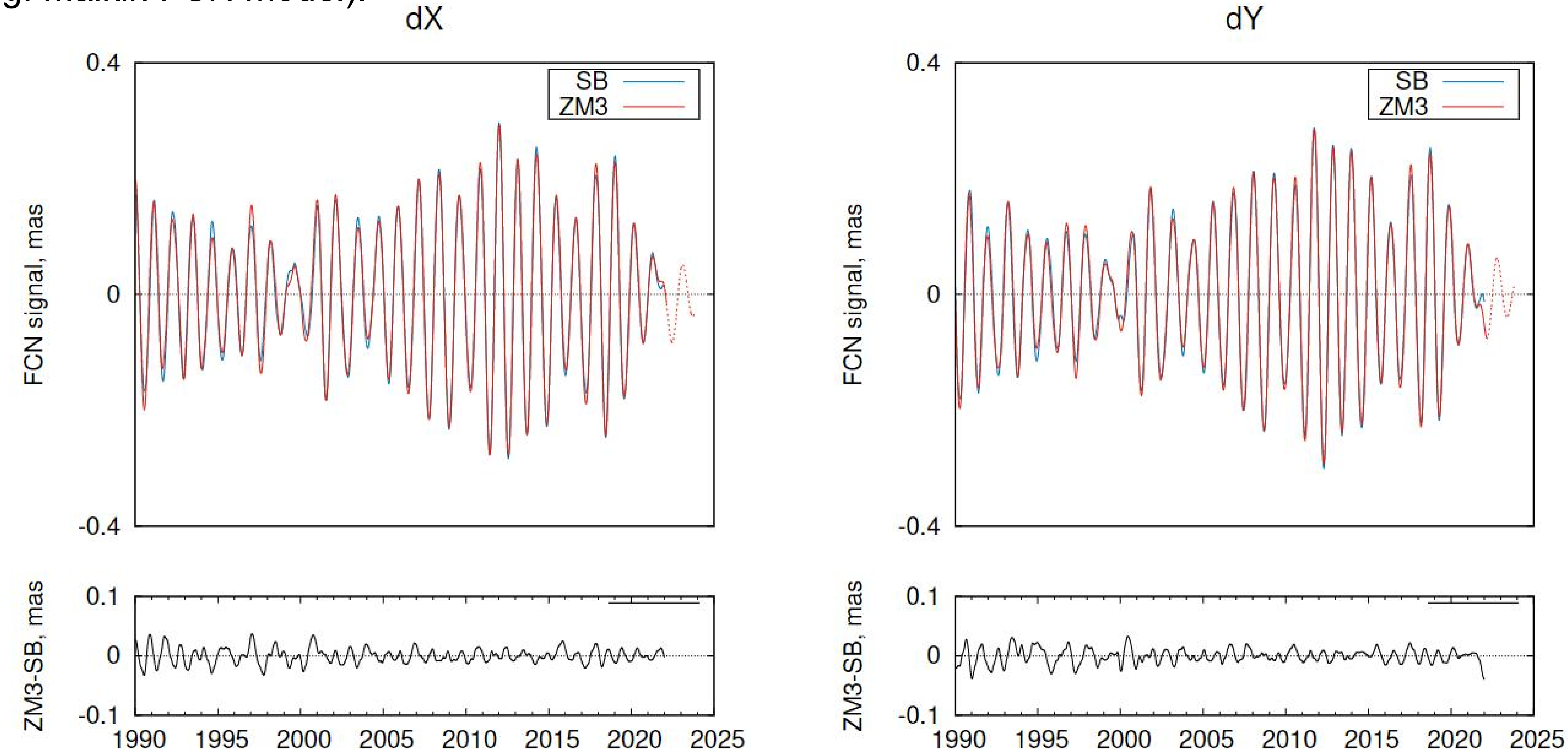
$$X_{\mathrm{VLBI}} = (X_{\mathrm{IERS/USNO}})_{\mathrm{apriori}} + dX$$
$$Y_{\mathrm{VLBI}} = (Y_{\mathrm{IERS/USNO}})_{\mathrm{apriori}} + dY.$$

$$X_{\mathrm{VLBI}} = (X_{\mathrm{IAU2006/2000A}})_{\mathrm{apriori}} + dX$$
$$Y_{\mathrm{VLBI}} = (Y_{\mathrm{IAU2006/2000A}})_{\mathrm{apriori}} + dY;$$

		Session Wise		Global solution	
		Std(dX)	Std(dY)	Std(dX)	Std(dY)
Standard	Case 1	192.3	188.9	107.7	112.4
	Case 2	185.5	183.5	88.6	91.5
N=100	Case 1	289.9	290.4	-	-
	Case 2	282.2	287.0	-	-
N=400	Case 1	295.1	286.4	111.1	113.9
	Case 2	286.2	282.2	92.46	93.1
N=800	Case 1	292.0	288.7	-	-
	Case 2	286.2	283.8	-	-

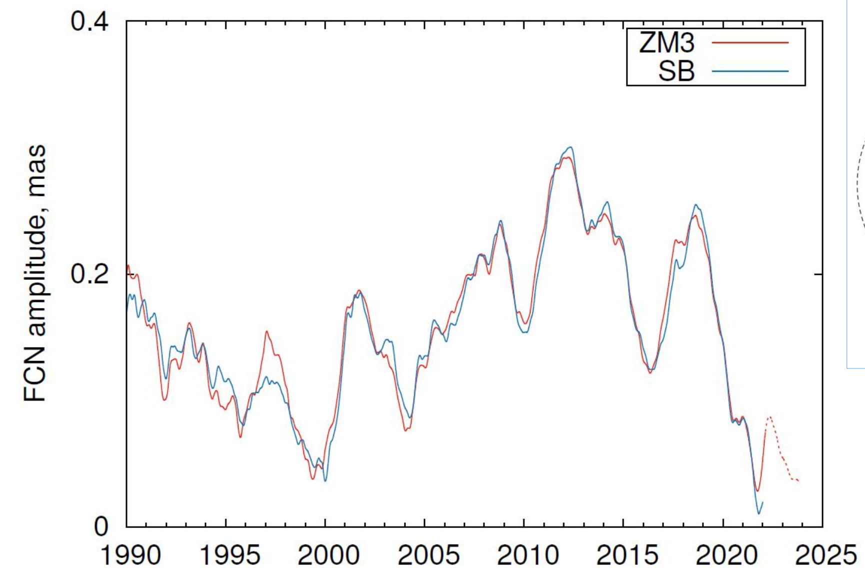
Free Core Nutation: Comparison w.r.t. Malkin FCN model

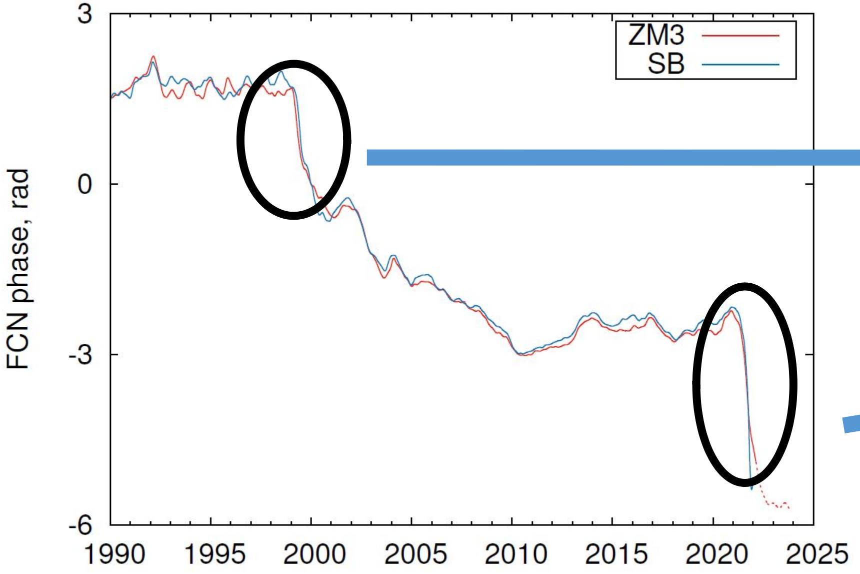
The parameterization of source coordinates leads to similar FCN models in comparison to the conventional ones (e.g. Malkin FCN model).

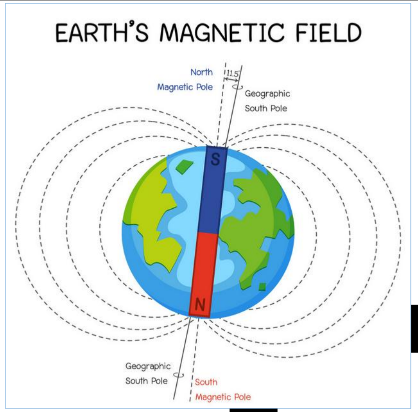


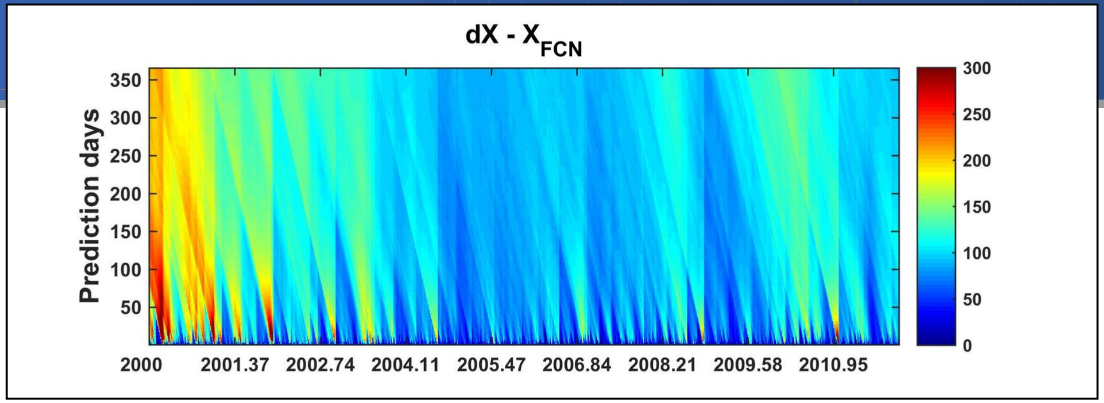
Upper panels: SB (blue line / this study) and ZM3 (red line/ Malkin) FCN series.
Bottom panels: ZM3 minus SB model differences

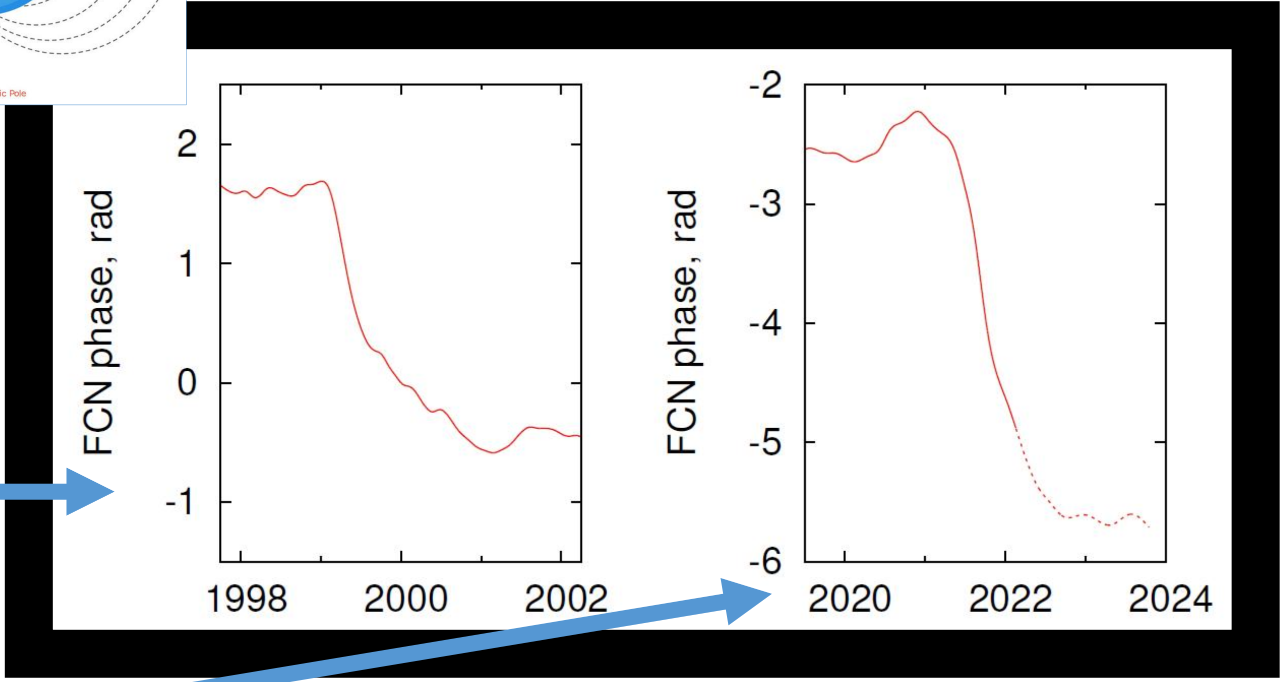
Variations of the FCN amplitude and phase





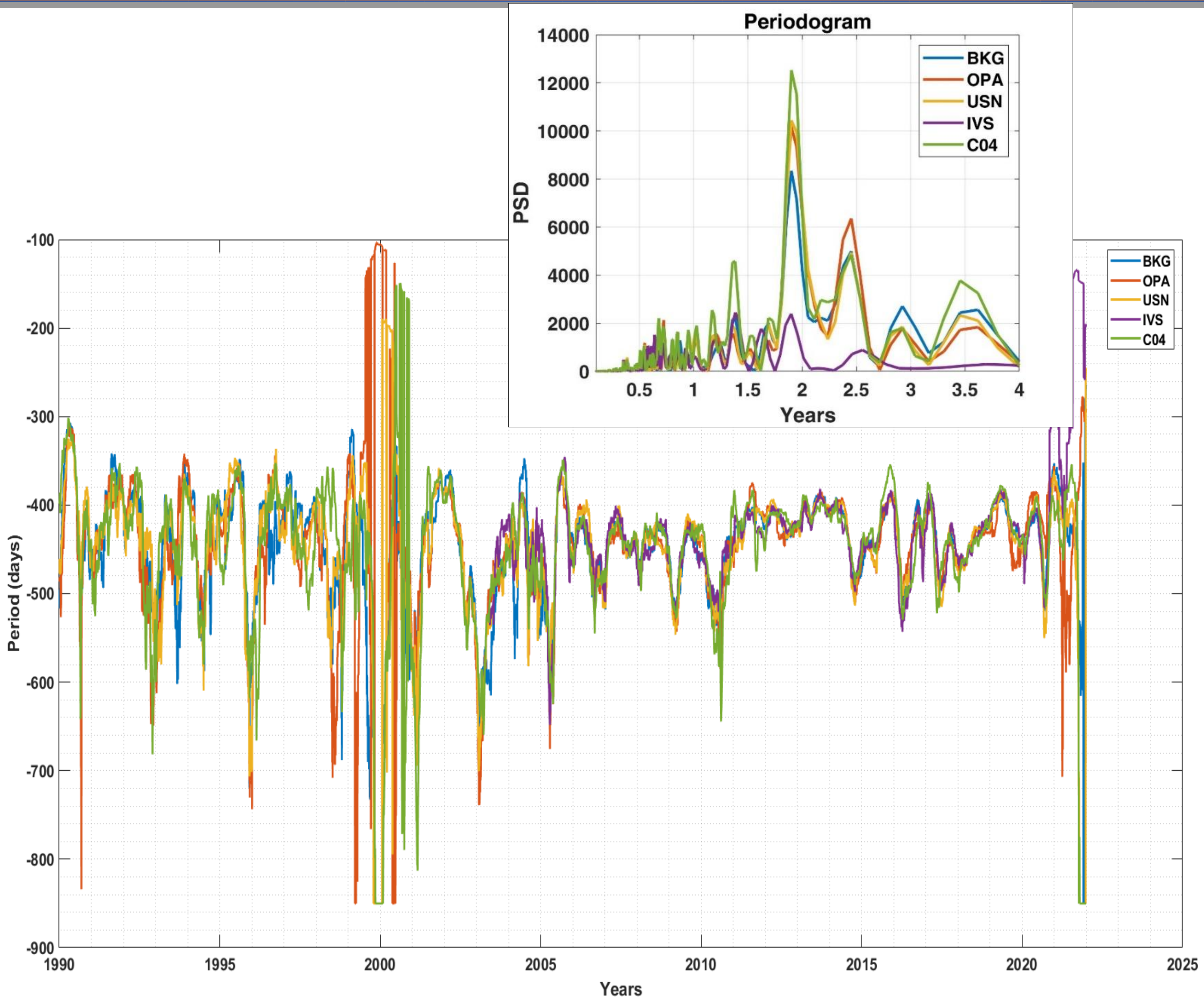






Malkin, Z.; Belda, S.; Modiri, S. Detection of a New Large Free Core Nutation Phase Jump. Sensors 2022, 22, 5960. https://doi.org/10.3390/s22165960

Optimum FCN period using a Sliding window of 400 days



Conclusions

- In this study, we estimated the CPO making use of different parameterization of source coordinates (i.e. MARS) and different VLBI strategies.
- The inclusion of MARS (Multi-adaptive regression splines algorithm) in the global adjustment is feasible and **improves slightly the accuracy of the precession offset and rate**.
- As expected, The **formal error analyses** evidence less accurate results using session-wise analysis.
- Using different approaches (global & session-wise / standard & MARS) **results in slight deviations**, particularly in the scatter of the CPO residuals.
- The empirical **corrections estimated in the reassessment of the precession and nutation terms attain an error reduction by almost 15 µas** (≈0.5 millimeters) for both the X and Y components.
- We found 12 frequencies with amplitude corrections larger than **3 times the median error (≈3 µas)**
- FCN empirical models agree with established ones.
-

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