Potential of Lunar Laser Ranging for the determination of Earth orientation parameters

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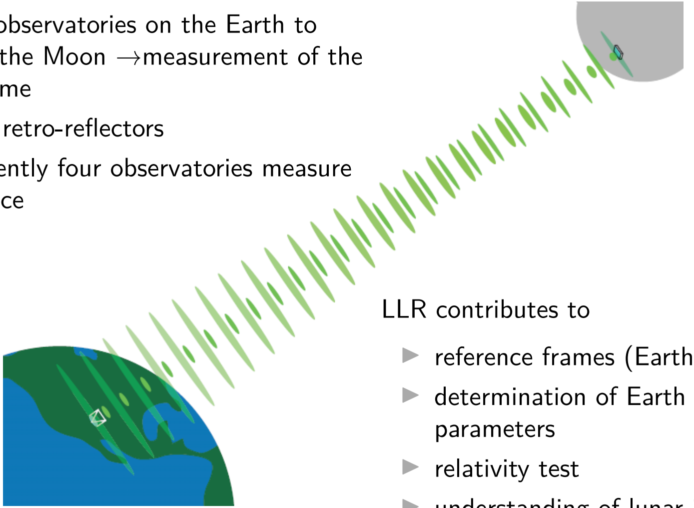
Principle of Lunar Laser Ranging

- Laser pulses from observatories on the Earth to retro-reflectors on the Moon →measurement of the round-trip travel time
- on the Moon: five retro-reflectors
- on the Earth: currently four observatories measure Earth-Moon distance



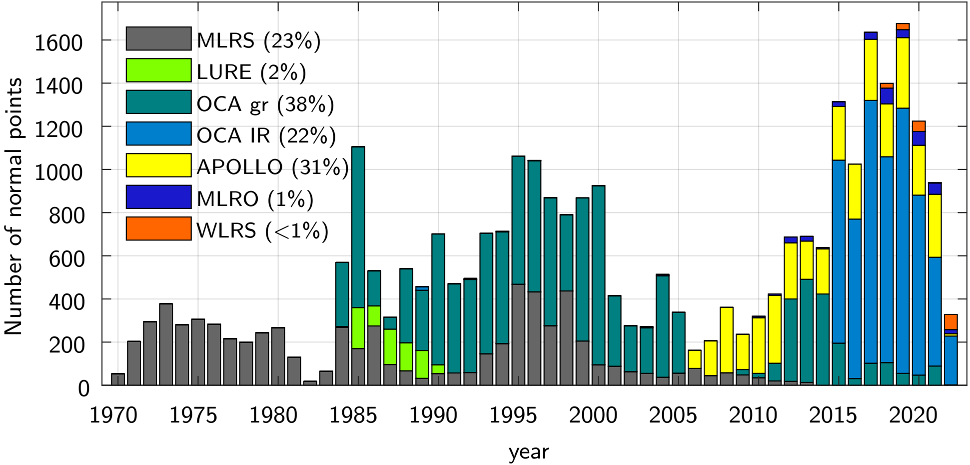
(Murphy, 2013)

LLR contributes to

- reference frames (Earth, Moon, inertial)
- determination of Earth orientation parameters
- relativity test
- understanding of lunar interior

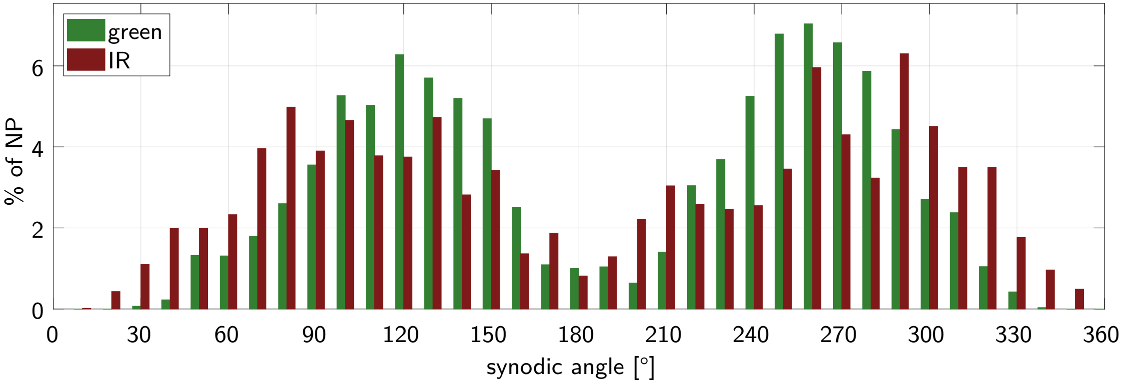
Distribution of the normal points over 52 years

30172 normal points over the time span April 1970 - April 2022



Distribution of the normal points over the synodic month

30172 normal points over the time span April 1970 - April 2022



LLR analysis at IfE

- iterative procedure between ephemeris calculation and parameter estimation
- initial positions and velocities of 8 planets, Sun, Moon, Pluto and asteroids (Ceres, Vesta, and Pallas) from DE440
- IERS Conventions 2010
- until 1983 use of the Kalman Earth Orientation Filter (KEOF) series COMB2019
- from 1983 IERS C04 EOP series
- up to 200 parameters can be determined
- as an extension, relativistic parameters can be determined

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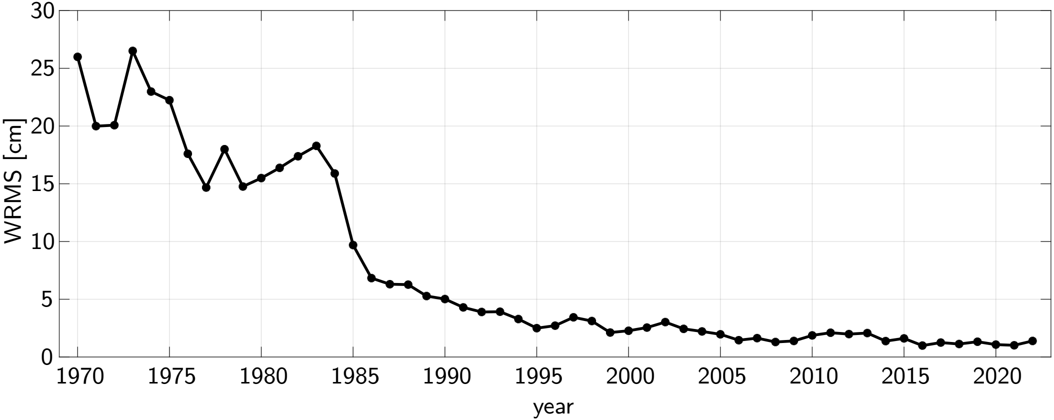
for the Earth

- station coordinates and velocities
- nutation coefficients and precession rate
- x_p, y_p and ΔUT (UT0 apart from VLBI)

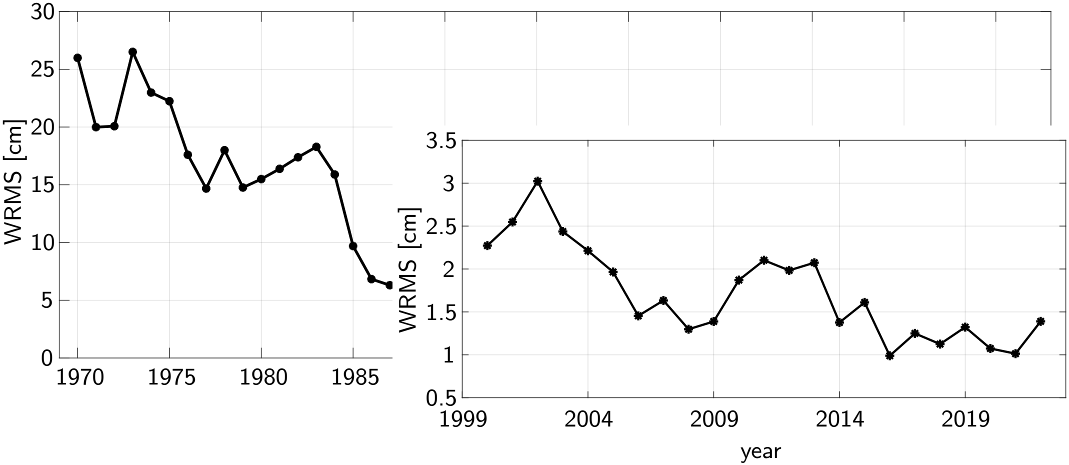
for the Moon

- initial values for orbit and rotation
- reflector coordinates
- dynamical flattening
- lunar core parameters and Love numbers

Residuals from LLR analysis at IfE



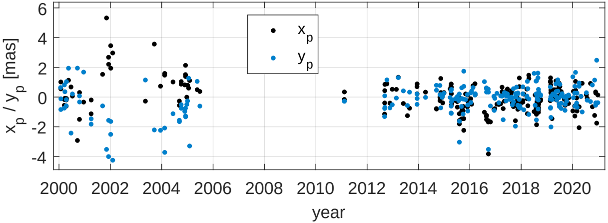
Residuals from LLR analysis at IfE



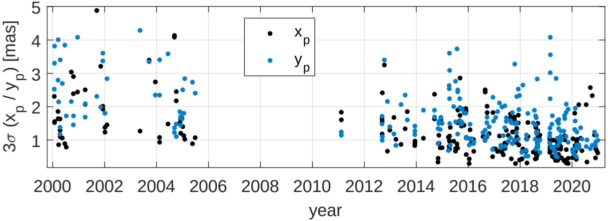
Earth rotation parameters from LLR

- all LLR NP are used to determine the parameters of Earth-Moon system
- pre-analysis to identify subsets of data with special conditions for ERP determination
- different constellations of stations and the number of NP per night tested
  $\Longrightarrow$ best results with LLR data from the Côte d'Azur Observatory, Grasse, France (OCA)
- simultaneous determination of either ΔUT1, x_p or y_p, coordinates of all observatories and other parameters of the Earth-Moon system
- velocities of the observatories fixed to ITRF2014 values
- a-priori ERP from IERS C04 series, fixed for those nights that were not considered
- 15 NPs per night for time span 01.2000 - 12.2020 (234 nights)
- on next slides: differences to the a-priori IERS C04 series, uncertainty $(3\sigma) = 3\times$square root of variance

x_p, y_p results



x_p, y_p differences to the a-priori IERS C04 EOP series

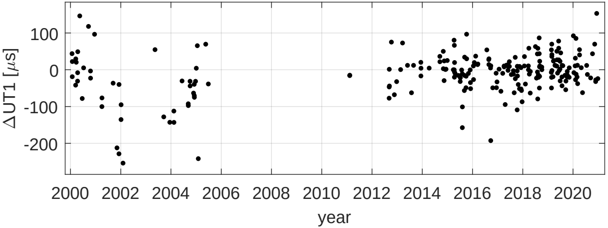


$3\sigma(x_p, y_p)$ uncertainties

- x_p values vary in the range of -3.8 mas to 5.3 mas with rms of 1.0 mas
- y_p values vary in the range of -4.2 mas to 2.5 mas with rms of 1.0 mas
- $3\sigma(x_p)$ values vary between 0.3 mas to 4.9 mas with rms of 1.4 mas
- $3\sigma(y_p)$ values vary between 0.5 mas to 4.3 mas with rms of 1.8 mas

(Singh et al. 2022)

ΔUT1 results



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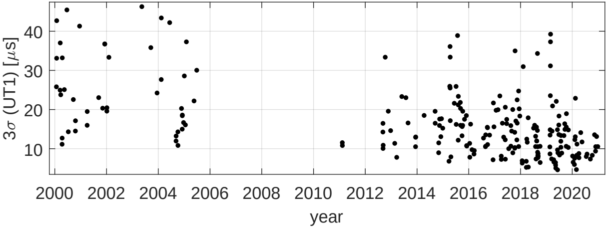

3σ(ΔUT1) uncertainties

- ΔUT1 values vary in the range of −253.8 μs to 153.1 μs with rms of 58.5 μs
- 3σ(ΔUT1) values vary between 4.6 μs to 46.3 μs with rms of 18.6 μs

(Biskupek et al. 2022, Singh et al. 2022)

Uncertainty discussion of ERP results

- uncertainty (σ) = square root of variance from least-squares adjustment
- because some small random and systematic errors remained in the LLR analysis, uncertainties were given as 3σ values in the past
- analysis of the factor for uncertainties: calculation of ERPs in different cases with different fixed parameters
  1. range = abs(abs(max(ERP))-abs(min(ERP)))
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ERP	case 1	case 2	case 3	case 4	range
x_p (mas)	0.56	0.56	0.57	0.57	0.21
y_p (mas)	0.64	0.64	0.57	0.57	0.26
ΔUT1 (µs)	5.66	5.76	5.79	5.90	17.92

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x_p (mas)	0.56	0.56	0.57	0.57	0.21	→ **0.6 mas** (1σ)
y_p (mas)	0.64	0.64	0.57	0.57	0.26	→ **0.7 mas** (1σ)
ΔUT1 (µs)	5.66	5.76	5.79	5.90	17.92	→ **18 µs** (3σ)

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- from VLBI: ΔUT1 (15 µs to 20 µs, intensive), x_p, y_p (50 µas to 80 µas)
- from SLR: x_p, y_p (10 µas to 30 µas), GNSS: x_p, y_p (5 µas to 20 µas)

Nutation results

period	results 2018 [mas]	results 2022 [mas]
$A_{18.6y}$	1.42 ± 0.53	0.48 ± 0.18
$B_{18.6y}$	-0.18 ± 0.19	-0.04 ± 0.09
$A''_{18.6y}$	-0.68 ± 0.37	0.38 ± 0.17
$B''_{18.6y}$	-0.06 ± 0.21	0.26 ± 0.10
$A_{9.3y}$	-1.12 ± 0.34	-0.23 ± 0.17
$B_{9.3y}$	-0.27 ± 0.15	-0.15 ± 0.07
$A''_{9.3y}$	-1.55 ± 0.34	0.60 ± 0.16
$B''_{9.3y}$	0.17 ± 0.14	0.13 ± 0.07
$A_{365.3d}$	1.05 ± 0.19	0.14 ± 0.10
$B_{365.3d}$	-0.51 ± 0.09	-0.05 ± 0.05
$A''_{365.3d}$	0.65 ± 0.15	-0.05 ± 0.09
$B''_{365.3d}$	0.04 ± 0.06	-0.09 ± 0.03

(Hofmann et al. 2018)

period	results 2018 [mas]	results 2022 [mas]
$A_{182.6d}$	0.51 ± 0.17	0.09 ± 0.08
$B_{182.6d}$	-0.06 ± 0.07	0.02 ± 0.04
$A''_{182.6d}$	-0.57 ± 0.14	0.18 ± 0.08
$B''_{182.6d}$	-0.07 ± 0.07	0.09 ± 0.04
$A_{13.6d}$	1.49 ± 0.63	0.39 ± 0.18
$B_{13.6d}$	-0.65 ± 0.26	-0.06 ± 0.08
$A''_{13.6d}$	-1.42 ± 0.81	0.12 ± 0.11
$B''_{13.6d}$	0.27 ± 0.32	-0.09 ± 0.05

- smaller differences to a-priori MHB2000 model compared to 2018 results
- uncertainties (3σ) improved by factor 2
- biggest improvement for 13.6d period, benefit from IR OCA data

Conclusion and outlook

- best LLR results for x_p, y_p and ΔUT1 from the highly accurate OCA data with 15 NPs per night
- determination of nutation coefficients with small differences to a-priori MHB2000 model and improved uncertainties
- new results benefits from IR NP with higher accuracy and better distribution over synodic month

next steps:

- uncertainty tests for nutation coefficients
- combination of VLBI and LLR for validation of EOP