Single band absolute VLBI astrometry

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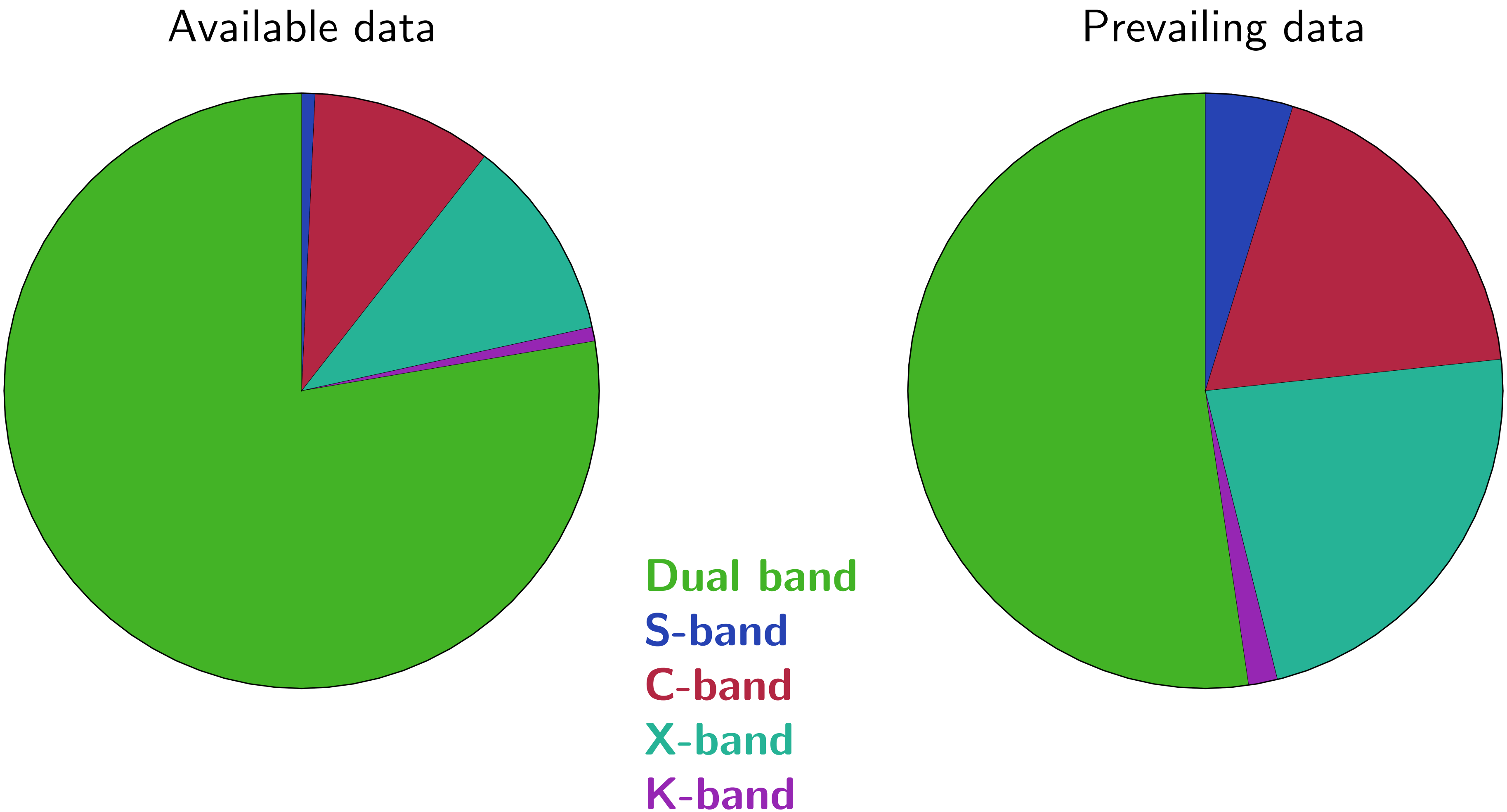
Motivation

Used bands of among 21,093 sources with VLBI absolute astrometry:



How does ionosphere affect single-band VLBI astrometry?

Problem statement:

Given:

- Observing campaign scheduled in one band
- Observing campaign scheduled in dual-band, but with some observations detected at one band only

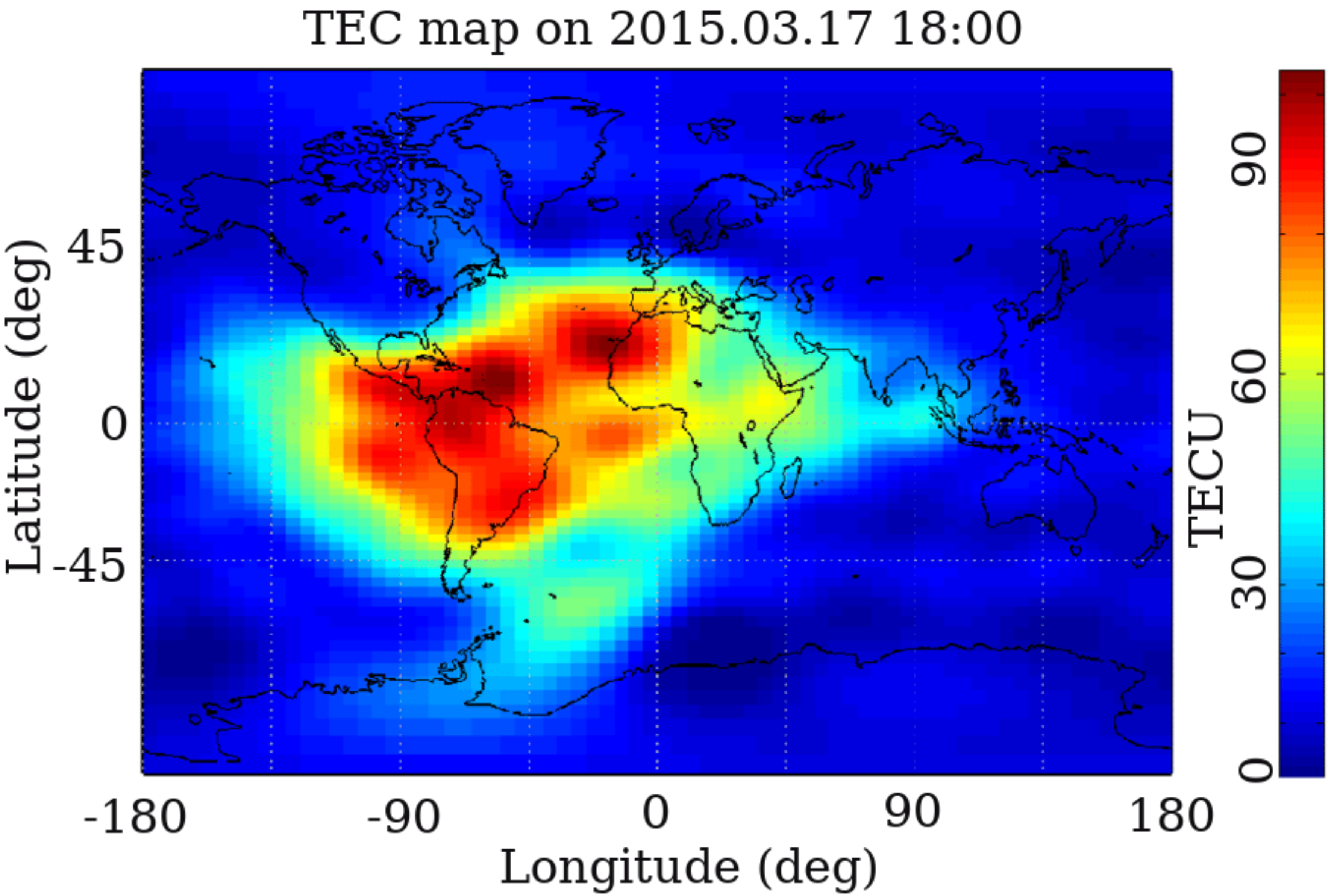
To find:

- data analysis algorithm that provide source positions with minimum uncertainties
- to provide *realistic* uncertainties source position from observations of both types

Motivation:

- to improve astrometry results
- to establish the baseline for investigation of ionospheric path delay from a 4-band VLBI

GNSS TEC maps provide an empirical ionosphere model in a form of 3D time series:



Case 1: dual-band observations with some obs only at a single band

Approach:

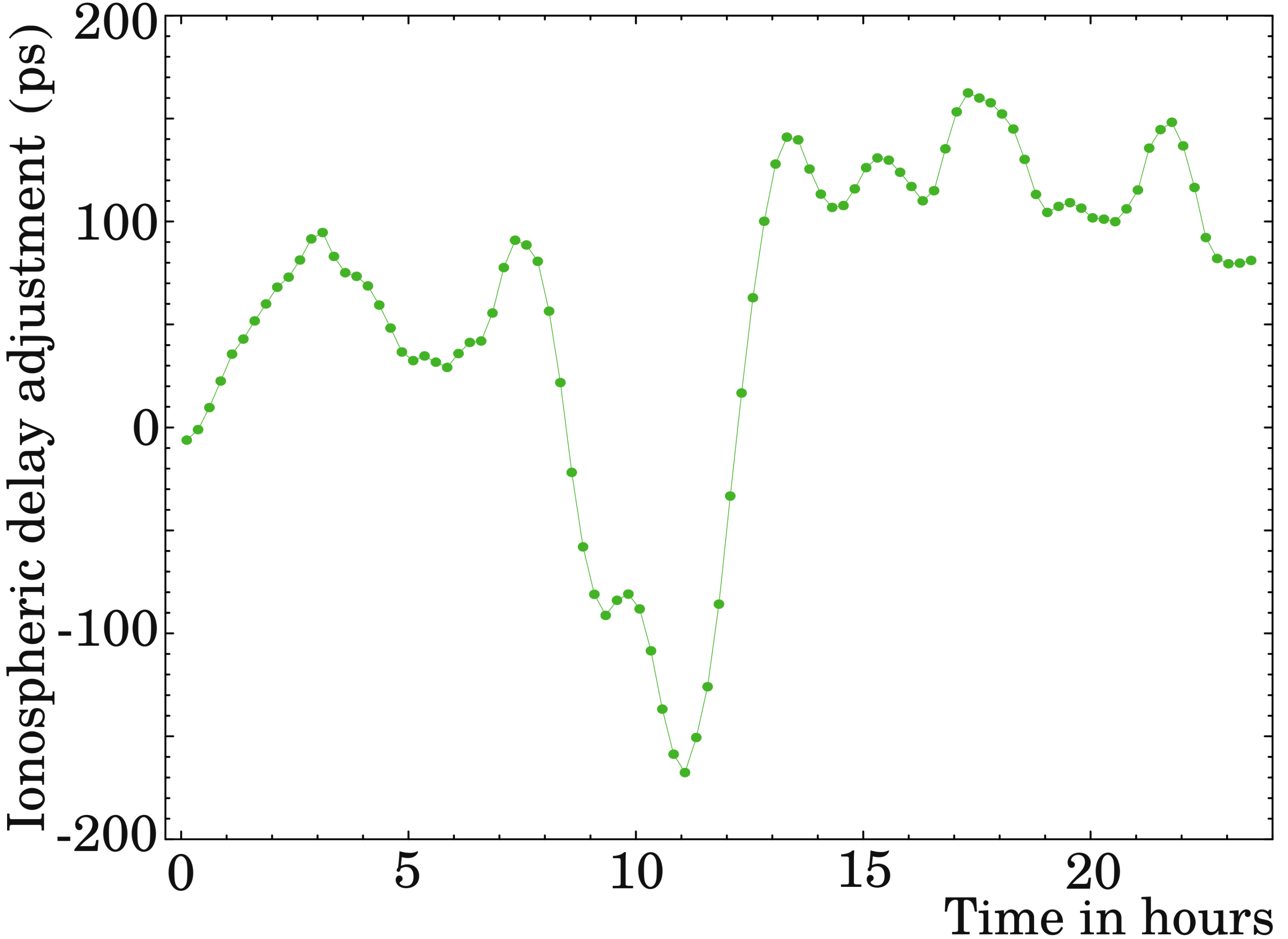
I represent ionospheric path delay at stations j, k as

$$\tau_i(t) = b_j(t) - b_k(t) +$$
$$\frac{\alpha}{f_e^2}\left(\left(\mathrm{TEC_j}(\phi_\mathrm{j}, \lambda_\mathrm{j}, \mathrm{t}) + \mathrm{a_j}(\mathrm{t})\right)\mathrm{M}(\mathrm{e_j}) \;-\; \left(\mathrm{TEC_k}(\phi_\mathrm{k}, \lambda_\mathrm{k}, \mathrm{t}) + \mathrm{a_k}(\mathrm{t})\right)\mathrm{M}(\mathrm{e_k})\right)$$

Both delay bias and TEC bias are expended in B-spline basis.

I seek coefficients a_{jk} and b_{jk} by fitting them into the ionosphere free combinations of group delays using LSQ

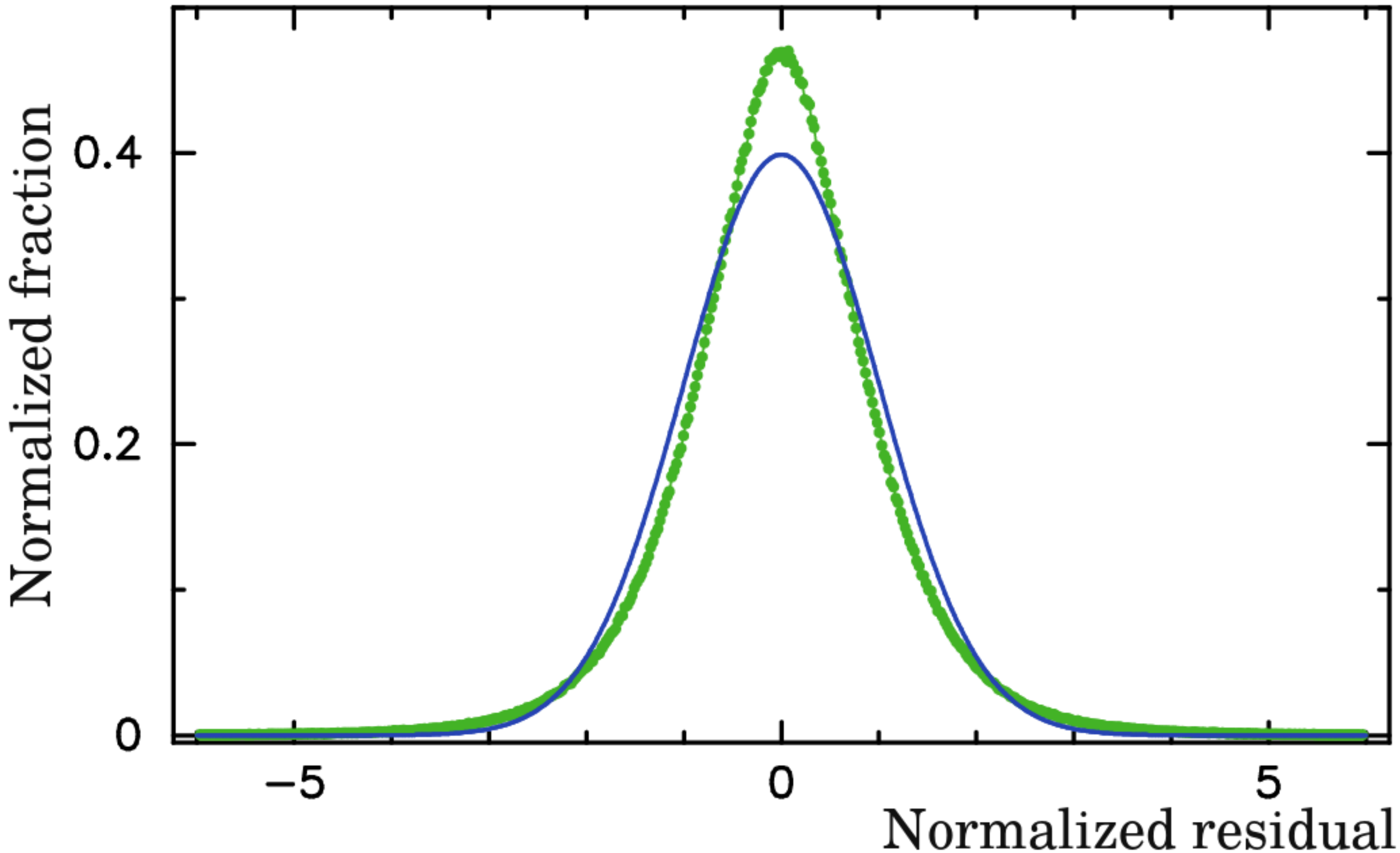
Adjustment to the ionosphere path delay bias at 8.4 GHz with respect to the path delay derived from GNSS TEC maps at MK-VLBA station from processing of dual-band observations on April 22, 2015.



Validation

- Took a dataset 4.3 million observations from 263 VLBA+ 2.3/8.6 GHz experiments in [1998, 2021], 5124 sources
- Computed a_{ij}, and b_{ij} using LSQ adjustment
- Computed τ_i, GNSS TEC maps + adjustment
- Computed $\sigma(\tau_i)$ using variance-covariance of a_{ij}, and b_{ij} estimates
- Computed $(\tau_i - \tau_{vi})/\sigma(\tau_i)$

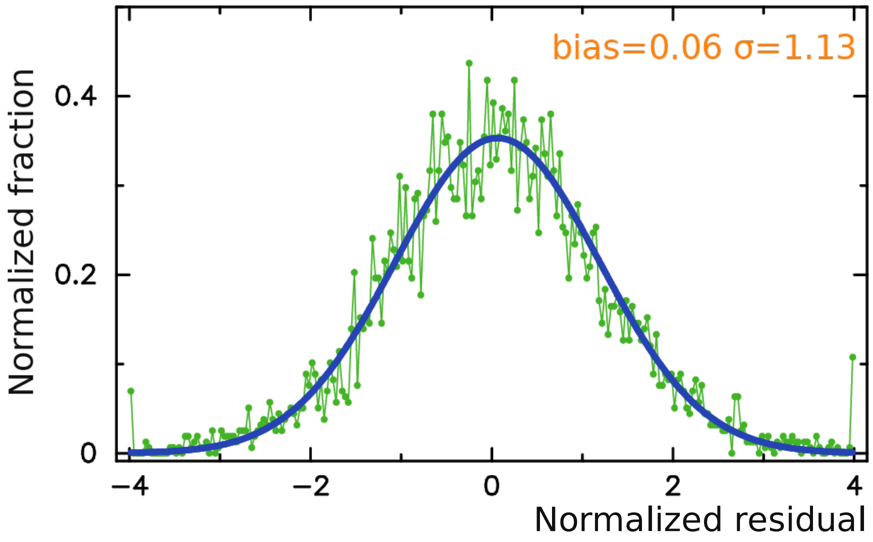
Empirical distribution of the normalized differences of the ionosphere path delay computed from the GNSS TEC maps adjusted for clock and TEC biases (green dots). The normal distribution with $\sigma = 1$ is shown as a reference (solid blue line):



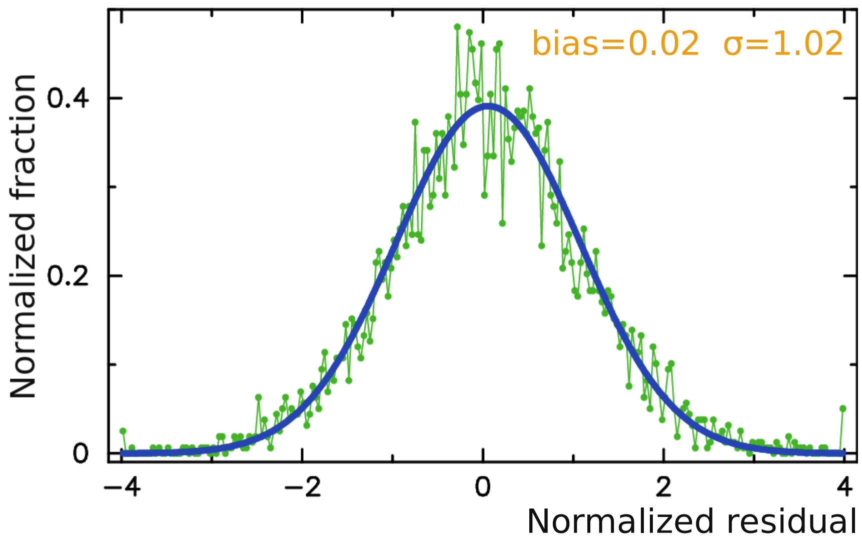
Distribution moments: 0.003 and 0.889

Distribution of differences with added post-iono residuals:

Post-iono residuals



Gaussian noise



parameter	post-iono		Gaussian noise	
	mean	σ	mean	σ
$\Delta\alpha$ X-band	-0.03	0.63	-0.01	0.56
$\Delta\delta$ X-band	0.02	0.64	0.01	0.56
$\Delta\alpha$ S-band	-0.05	1.14	-0.01	1.03
$\Delta\delta$ S-band	0.06	1.13	0.02	1.02

Case 2: Single band observing sessions

Let us take dual-band data, compute ionospheric delay from GNSS TEC maps, and compute residual ionospheric path delay

$$\tau_r = (\tau_{\rm gi} - \tau_{\rm vi} - c_{jk})/\tilde{M},$$

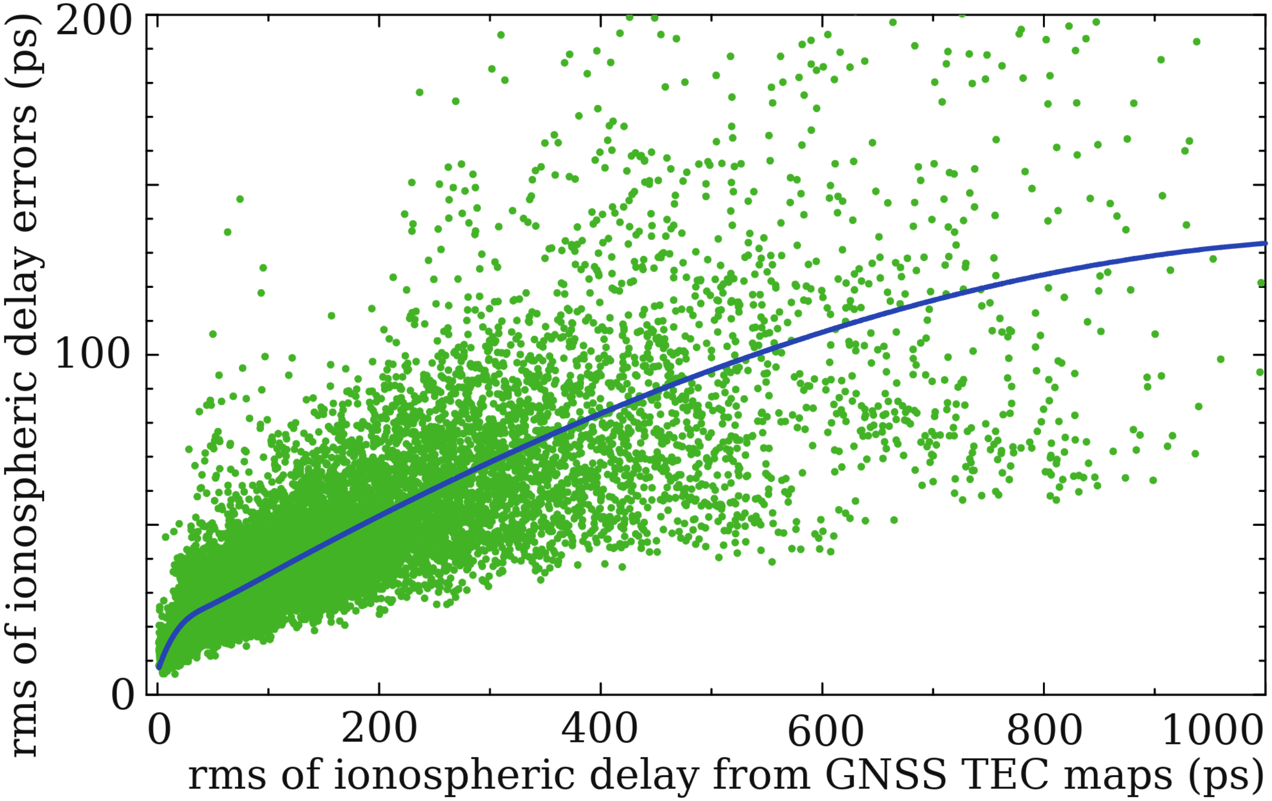
where $\tilde{M} = (M(e_1) + M(e_2))/2.0$ and c_{jk} is the clock bias.

Then I search for a regression model for σ_{τ_r}.

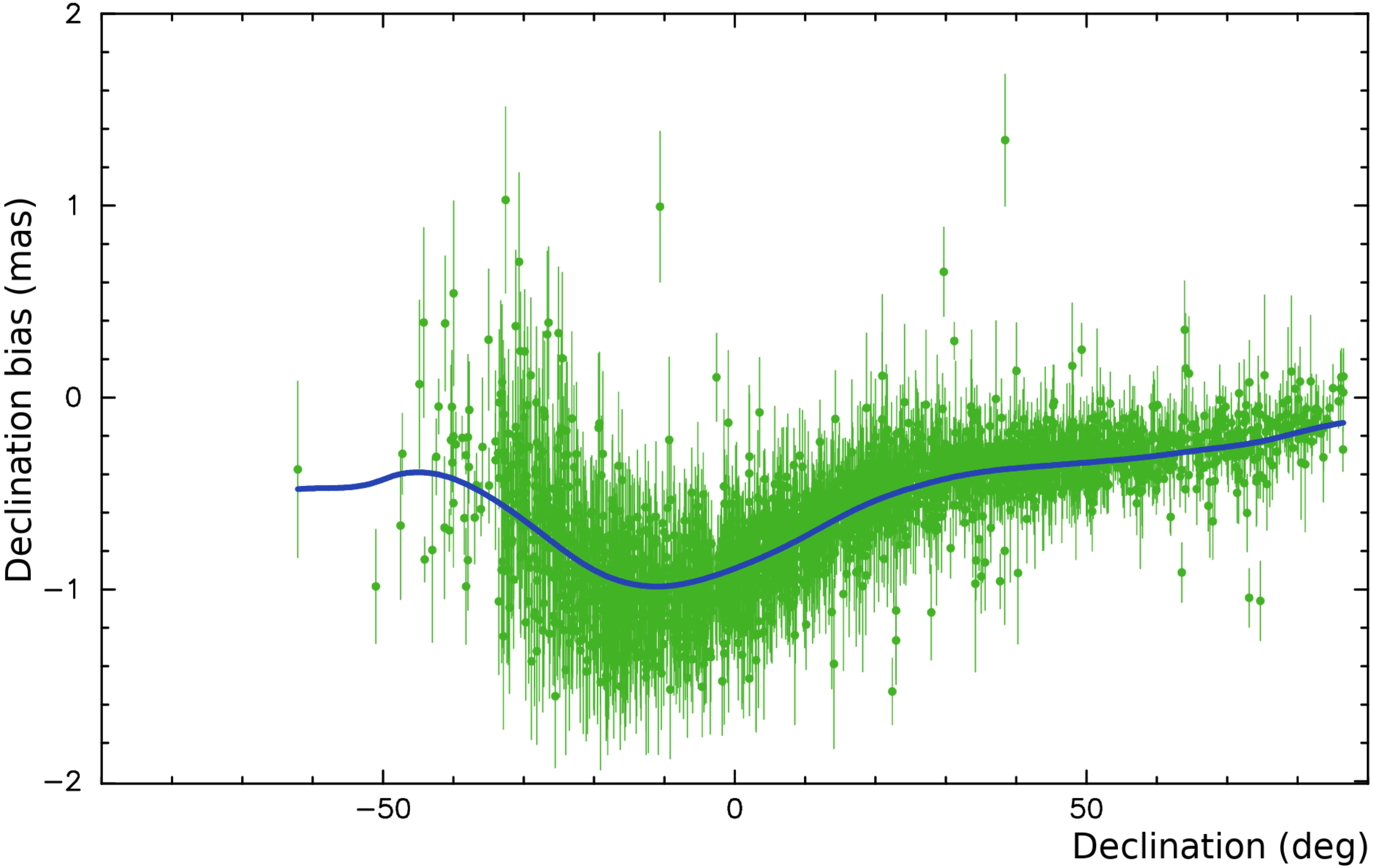
I took $\sigma_{\tau_{\rm gi}}$ as a regressor.

I anticipated a power law $\sigma_{\tau_{\rm r}}$ dependence on $\sigma_{\tau_{\rm gi}}$

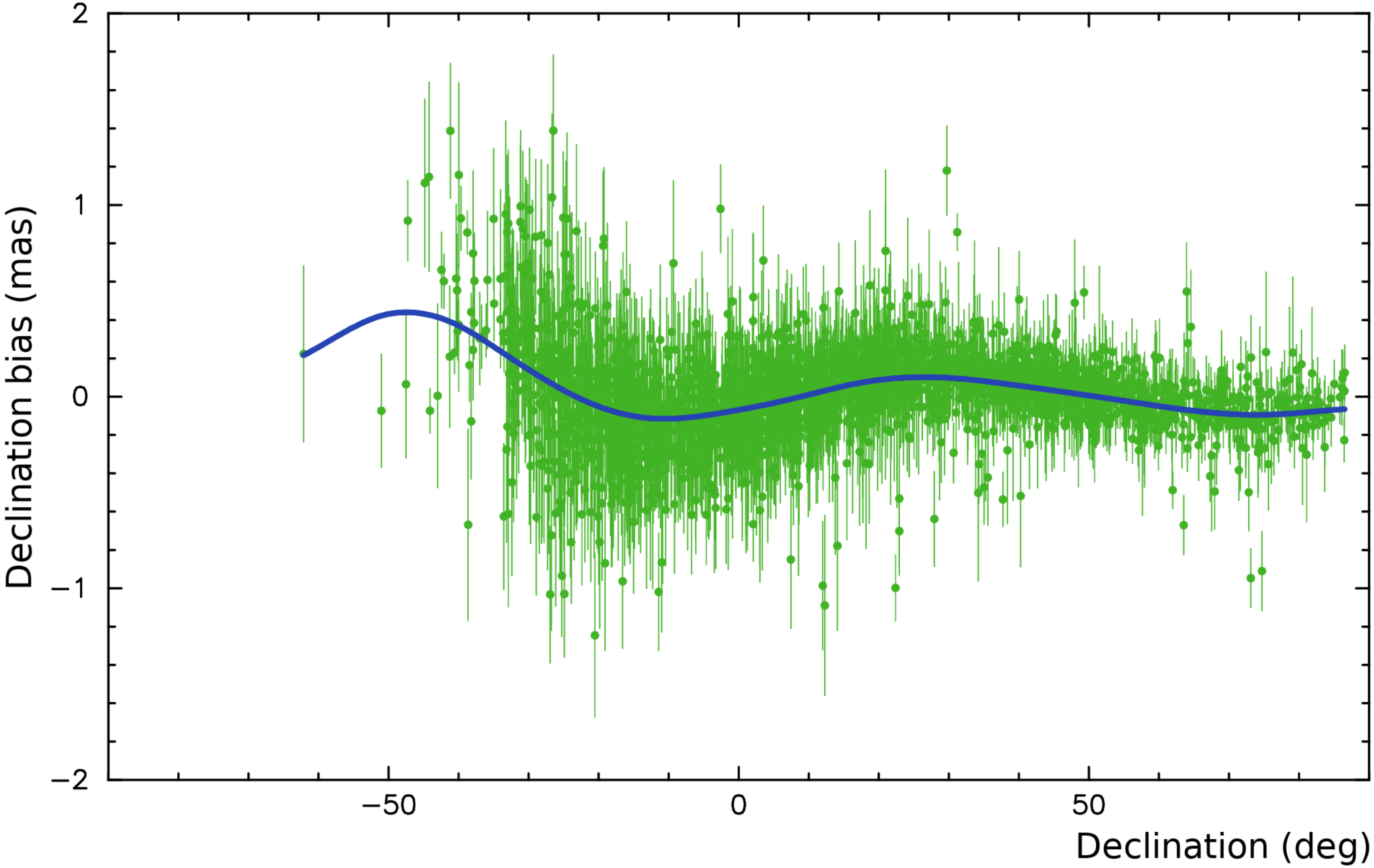
Dependence of the rms of residual ionospheric path delay derived from GNSS TEC maps on the rms of the total ionospheric path delay from these maps. No adjustment to TEC has been applied. Path delay is computed for the reference frequency 8 GHz. The blue smooth line shows the regression model in a form of a B-spline that fits the data.



A significant declination bias in X-band solution was detected:

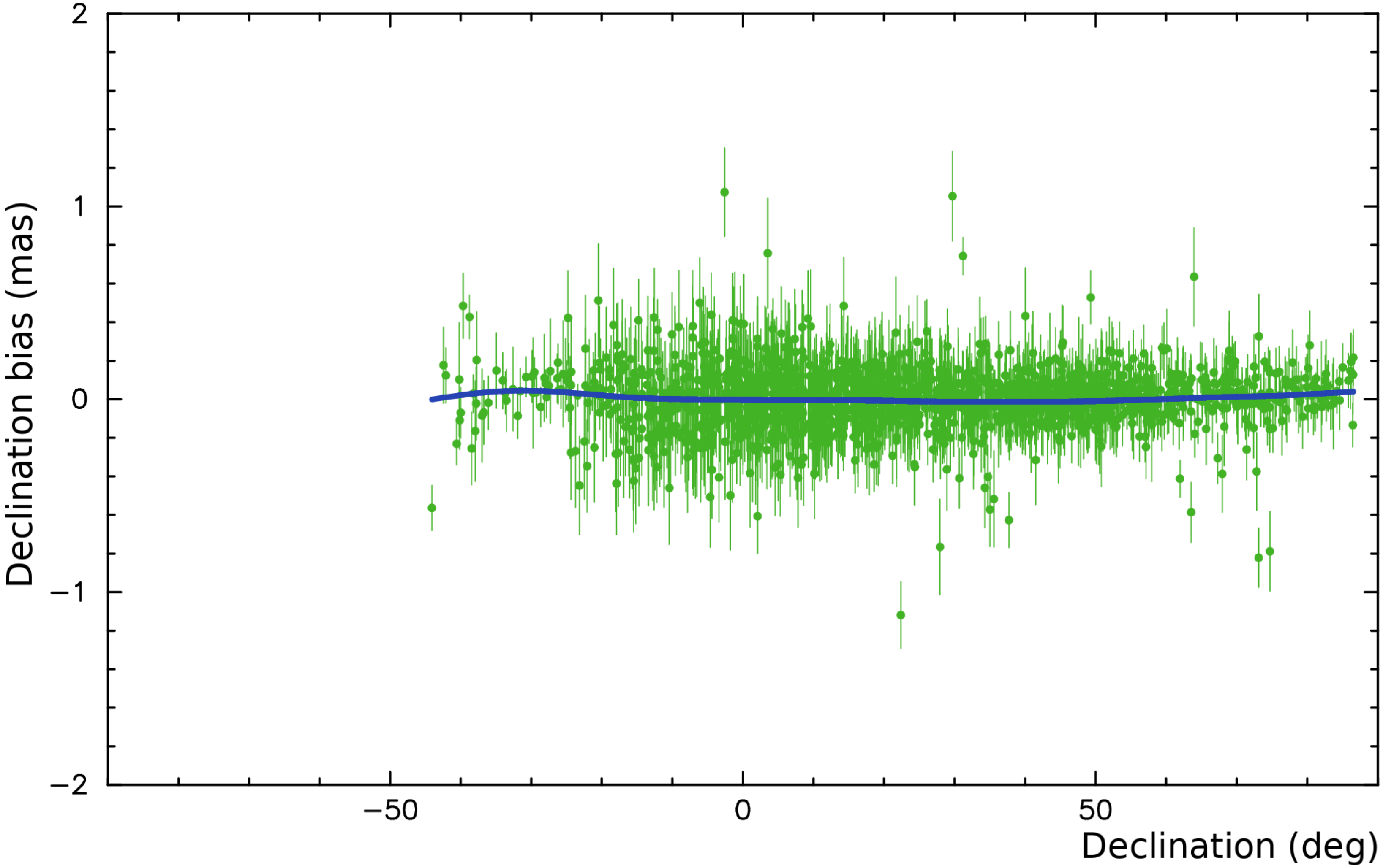


Scaling the ionospheric contribution by 0.772 significantly reduced bias!



Next step: develop an empirical bias model $b(\delta)$ and add the contribution in delta reduction: $\frac{\partial\tau}{\partial\delta}\, b(\delta)$

This drives the bias to a negligible level:



Modifying mapping function

JPL modified ionospheric mapping function

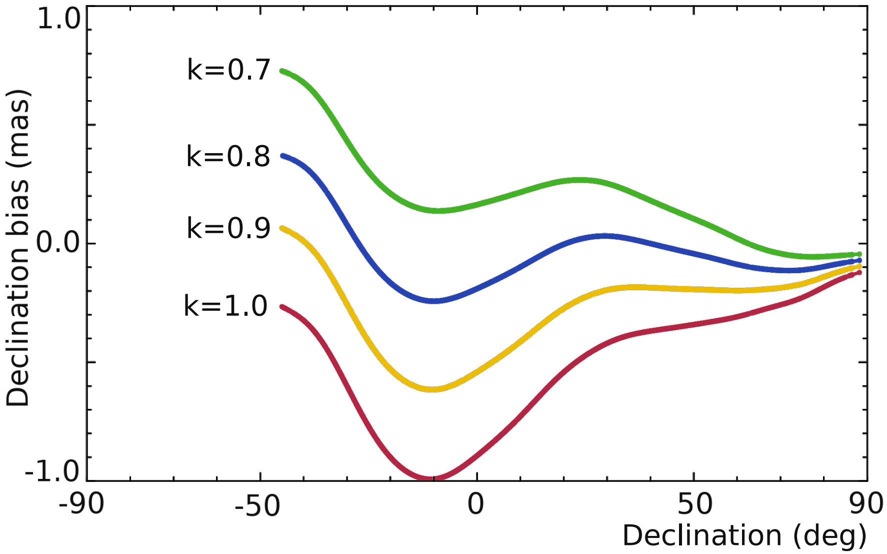
$$M(e) = \frac{1}{\sqrt{1 - \left(\frac{\bar{R}_{\oplus}}{R_{\oplus} + H_i + \mathbf{\Delta H}} \right)^2 \cos^2 \alpha e_{\mathrm{gc}}}},$$

I ran solutions with two modified mapping function:

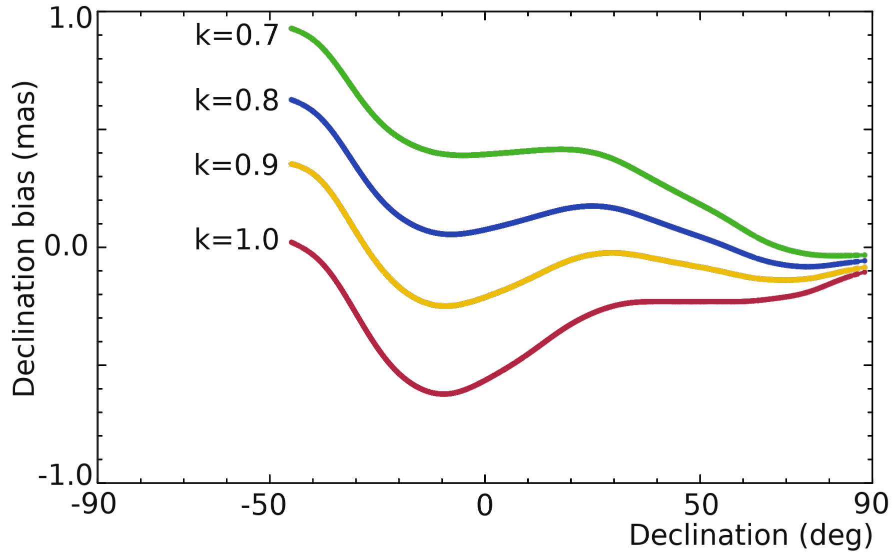
- Case 0: $\Delta H = 0.0$ km, $\alpha = 1.0000$
- Case 1: $\Delta H = 56.7$ km, $\alpha = 0.9782$
- Case 2: $\Delta H = 150.0$ km, $\alpha = 0.9782$

Smoothed declination biases for different mapping functions

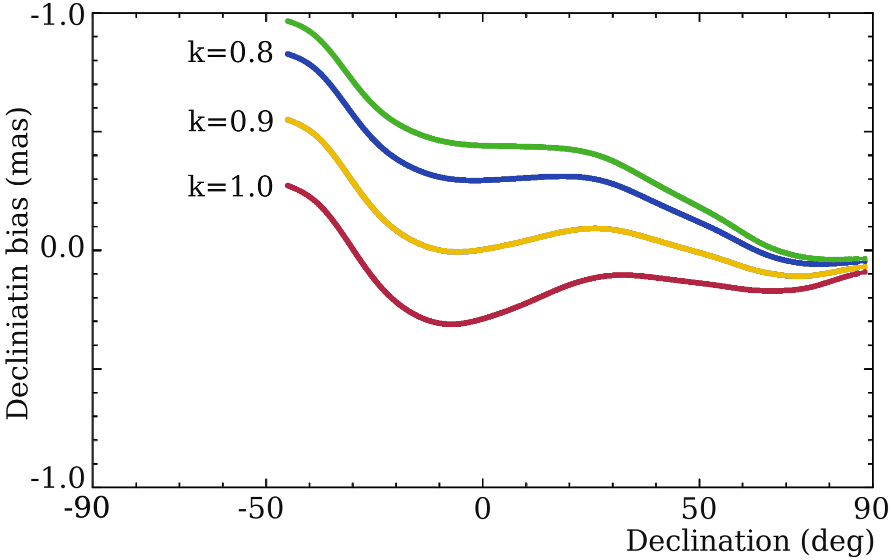
Case 0 $H_i = 450$ km, $\alpha = 1.0$



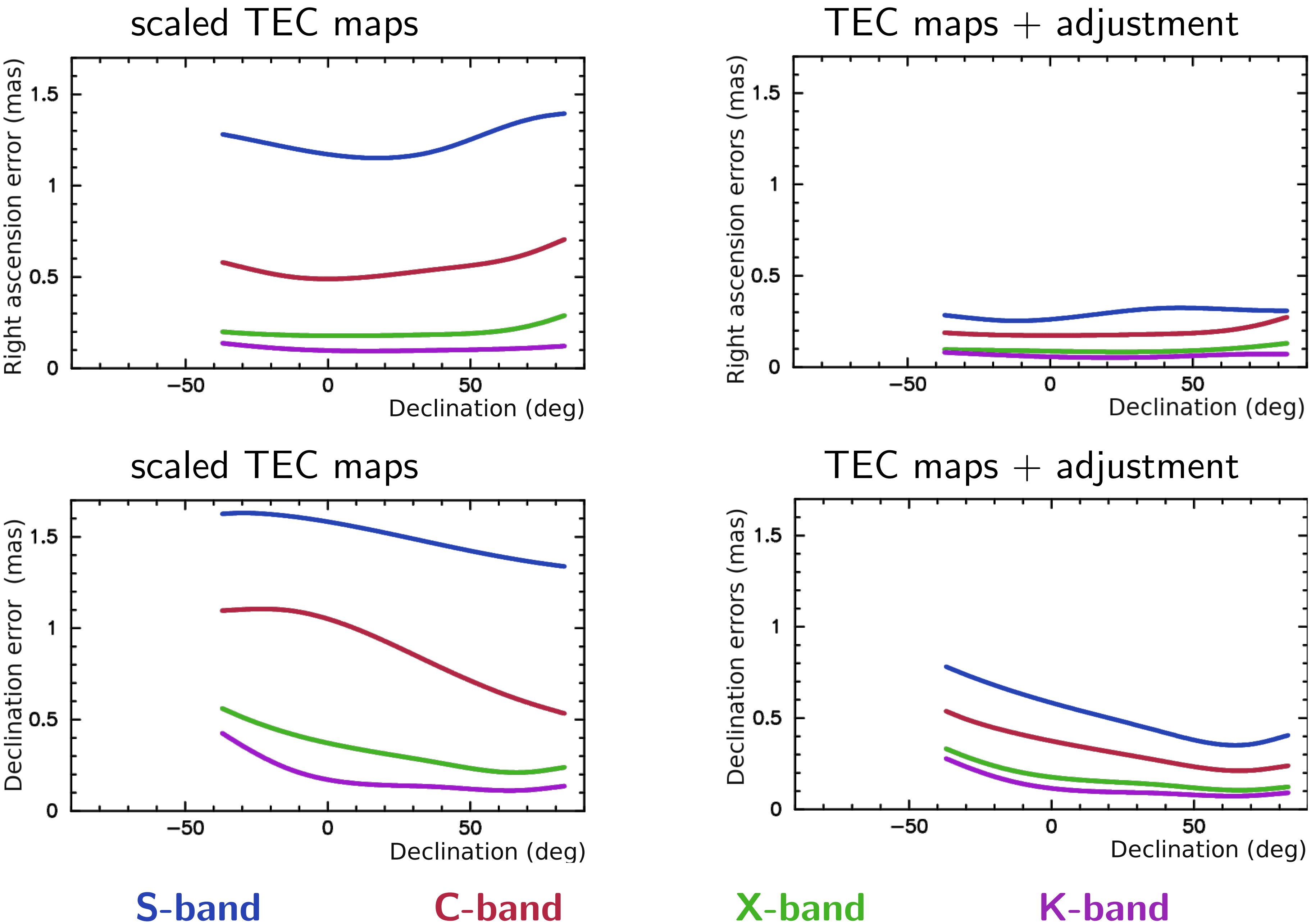
Case 1 $H_i = 506.7$ km, $\alpha = 0.9782$



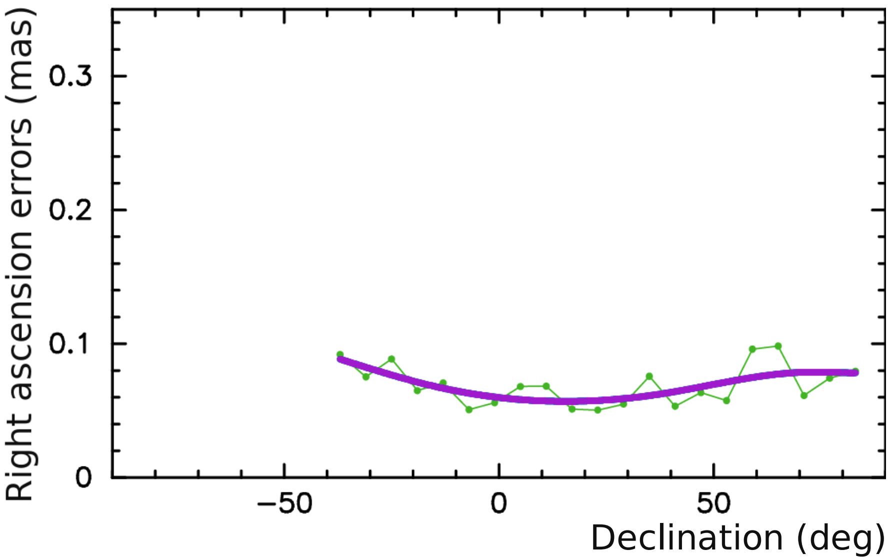
Case 2 $H_i = 600$ km, $\alpha = 0.9782$

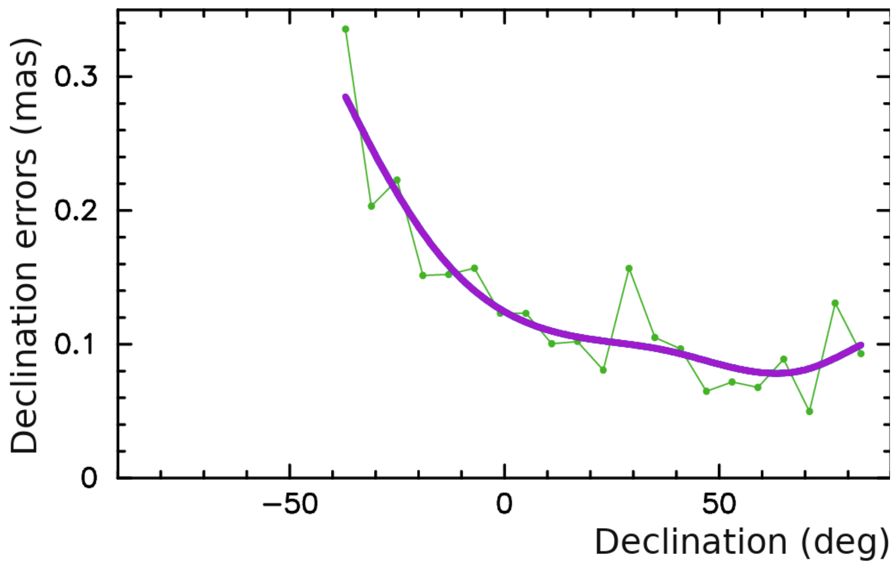


Extra noise in α, δ in single-band astrometry due to ionosphere



Extra noise in α, δ in K-band astrometry due to ionosphere





Summary:

- Adjusting delay and TEC bias using existing dual-band data provides superior results with no bias and with known additive noise

- Using single- and multi- band data from the same experiment in a single LSQ is feasible and does not introduce biases

- Using GNSS TEC maps *only* provides unsatisfactory results. Most likely reason: errors in mapping function.

- The following alleviates biases and improves realism of reported uncertainties:
  - scaling TEC by 0.85
  - using modified mapping function
  - applying a declination bias in data reduction
  - re-scaling additive weights

- Residual ionospheric errors at K-band introduce a noise 0.1–0.2 mas in source positions