Strategies for the optimal combination between local 3D modern GNSS and 2D classical networks, expressed in different reference frames: Case study in Greece

D. Ampatzidis[1], E. Tzanou[1], N. Demirtzoglou[2], G.S. Vergos[3]

[1]Assitant Professors, International Hellenic University, Dep. Surveying and Geomatics Engineering
[2]MSc. Dipl. Surv. Eng., Freelancer
[3]Professor, Aristotle University of Thessaloniki, Dep. Of Geodesy and Surveying

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Presentation's guidelines

1. Problem's statement

2. Existing methodologies

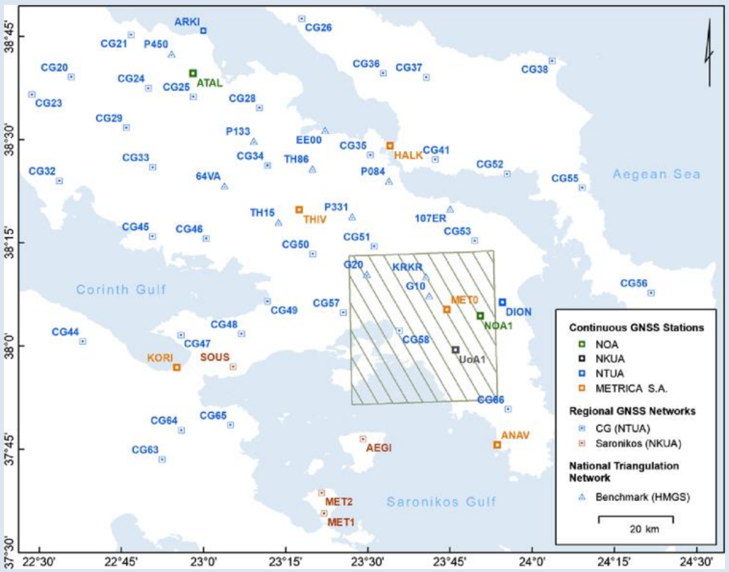
3. Theoretical considerations

4. Treatment of the problem through different scenarios

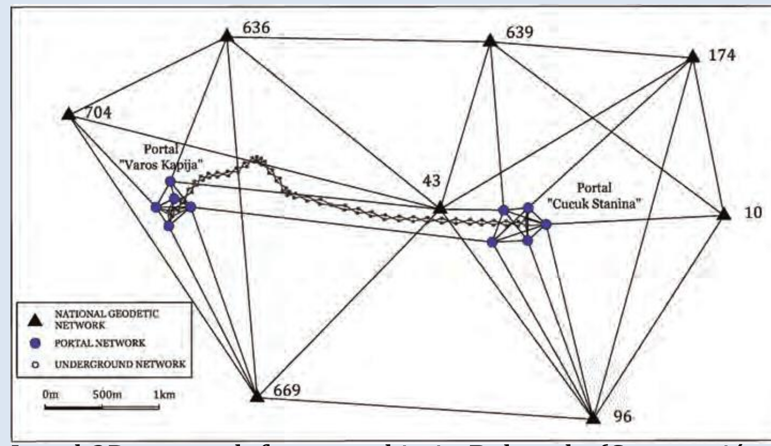
5. Case study

GNSS networks

With the advent of GNSS, one of Geodesists/Surveyors “dreams” came true: GNSS solve many problems in the field: quick, accurate and 3D information.



Local GNSS network in Attica
Foumelis (2019), https://doi.org/10.1515/jag-2019-0012



Local 2D network for tunnel in in Belgrade (Savanović et al. 2015, 10.15292/geodetski-vestnik.2015.03.564-576)

However, till today, the majority of the geodetic infrastructure (networks, traverses) remains in 2D. The historical data can not (and they should not) be abandoned and “disposed” .

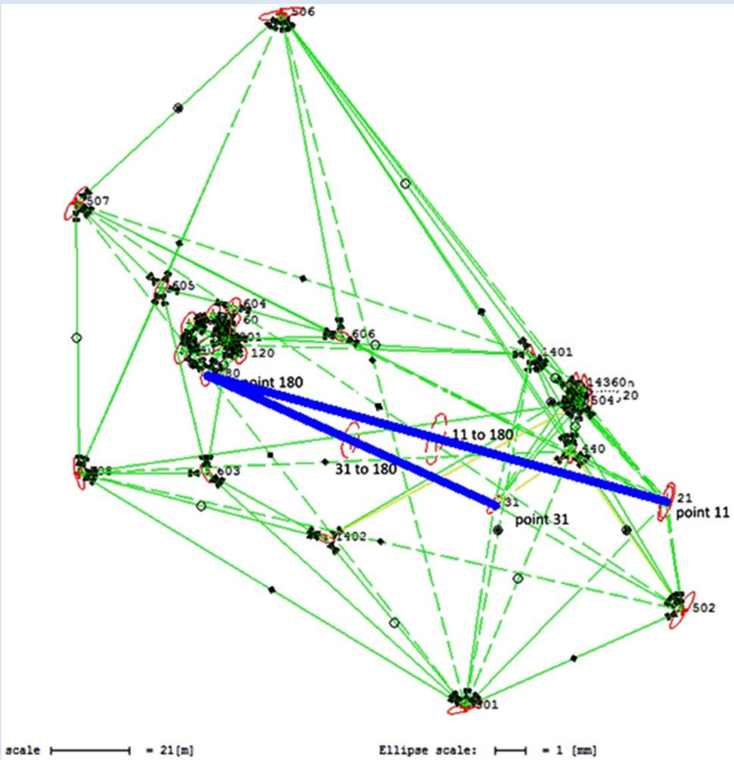
Need of 3D and 2D networks combination

The combination between 3D and 2D geodetic networks studied in depth by many researchers and technicians. The main scope of the combination is to align the combined network to a unified reference frame (to a modern TRF→ wished).

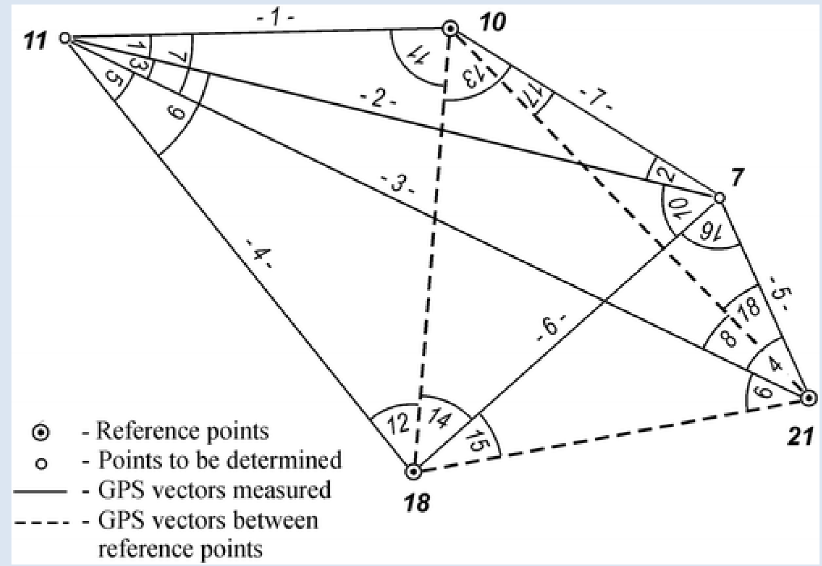
Two are the most used methodologies:

a. **Common Adjustment (CA): The GNSS observations are introduced as 3D baselines (dX, dY, dZ) and the 2D/1D observations as they are observed.**

b. **Helmert transformation (HEL): Through common points, one set of stations is transformed to the other's reference frame.**



A complete geodetic network (3D+2D+1D obs, local tie at Metsähovi ITRF station). Niemeier and Tengen (2017), https://doi.org/10.1515/jag-2016-0017



A complete geodetic network Gargula(2010), https://doi.org/10.1061/(ASCE)SU.1943-5428.0000042

Some considerations….

CA Method

A common adjustment, mostly in 3D (reliable existing softwares).

Some issues:

a. Using of 3D baselines:

What about having a **SINEX** file instead of baselines?
How the baselines are computed?
What about their full CV information?
What about their dependence on the reference frame (constraints)?

b. **Using of zenith angles**: Are they corrected to be consistent to the 3D information w.r.t. a geodetic system (Deflections of the Vertical-DoV)?

c. Normally, the CA method is conducted to a well-designed network with good geometry. **What about to analyze independent networks (designed for different purposes)?**

HEL Method

Widely used (sometimes abused..). Two sets of points: common/non-common

Some issues:

a. Poor geometry
What about the case of existing common points which do not enclose the area?

b. **Strong correlations** among parameters, especially for a small area→ low parameters quality

c. When 2D information is used, the up component is unavoidably vanished→ **loosing the vertical information**

Conclusion

Though the existing methods of combination are widely and successfully used, they have some pitfalls which maybe lead to less accurate results

New approach

Aim

To build rigorous algorithms in order to combine 3D (GNSS) networks which is given in a SINEX file and 2D classical network (spatial distances, horizontal and zenith angles and/or azimuths). The new approach focuses:

1. The exploitation of both 3D and 2D full information
2. Reference System explicit definition
3. Different scenarios, according to the needs and the accuracy of the existing observations

Key role to our approach play the **observed zenith angles and the deflections of the vertical**, as we will point out next.

Zenith angles are firstly corrected (refraction, curvature of the earth)

New approach : Steps

Common steps for all scenarios

1. Transformation of the 3D geocentric coordinates to the topocentric (ENU) system (SINEX file). This holds for both the approximate and estimated coordinates . The 3D network refers to a modern 3D TRF.

2. Transformation of the CV matrix of the 3D network from geocentric to topocentric system $\mathbf{C}_{\mathbf{x}}^{\mathbf{3D}} \rightarrow \mathbf{C}_{\mathbf{q}}^{\mathbf{3D}}$ where $\mathbf{q}$ the vector of the topocentric coordinates.

3. Inversion $\mathbf{C}_{\mathbf{q}}^{\mathbf{3D}} \rightarrow \mathbf{N}_{\mathbf{q}}^{\mathbf{3D}}$ and estimation of the Right Hand Side vector (RHS) $\mathbf{u}_{\mathbf{q}}^{\mathbf{3D}} = \mathbf{C}_{\mathbf{q}}^{\mathbf{3D}}\mathbf{q}$

4. Through the approximate topocentric coordinates of the common points between 3D and 2D networks, the new approximate values of the 2D network are re-calculated (analytical geometry) . <u>Now, the 2D network refers to the modern 3D TRF</u>

Scenario 1: Transition from 2D to 3D network through the zenith angles

Normally, the classical network includes observations of **zenith angles** refer to the physical surface (plumb line).
Through the Deflections of Vertical (DoV), they refer to an ellipsoid (geodetic system). (Rossikopoulos 1999, Barzaghi et al. 2016)

The observations of the classical network are: spatial distances, horizontal angles, zenith angles.

Steps (continued):

5. Combination of GNSS and classical network in full 3D (NEQ stacking, inverse variances act as weights, CDR method, Kotsakis and Chatzinikos 2017)

6. Restore the information from topocentric to geocentric

Major issue: the accuracy of the zenith angles and the knowledge of the DoV

Corrections and reduction of the measured zenith angle (plumb line)

$$z_{ij} = \underbrace{\zeta_{ij}}_{observed} + \underbrace{\frac{k_{ij}}{2R}\rho_{ij}\sin\zeta_{ij}}_{refraction} - \underbrace{\frac{1}{2R}\rho_{ij}\sin\zeta_{ij}}_{Earth's\ curvature} + \underbrace{\sin a_{ij}\eta_i + \cos a_{ij}\xi_i}_{DoV}$$

Observations' analytical geometry

$$\delta_{ij} = \arctan\frac{\Delta E_{ij}}{\Delta N_{ij}} - \theta_k$$

direction (θ orientation constant)

$$\omega_{ijk} = \arctan\frac{\Delta E_{ik}}{\Delta N_{ik}} - \arctan\frac{\Delta E_{ij}}{\Delta N_{ij}}$$

horizontal angle

$$\rho_{ij} = \sqrt{\left(\Delta E_{ij}\right)^2 + \left(\Delta N_{ij}\right)^2 + \left(\Delta U_{ij}\right)^2}$$

spatial distance

$$z_{ij} = \arctan\frac{\sqrt{\left(\Delta E_{ij}\right)^2 + \left(\Delta N_{ij}\right)^2}}{\Delta U_{ij}}$$

zenith angle

Scenario 2: Transition from 2D to quasi-3D network through the zenith angles

This scenario is practically a sub-case of the 1st one: In order to mitigate -as possible- the effect of the lower accuracy of the zenith angles (wrt the other observables), we split the classical network to.

a. Classical solution of 2D network → $\mathbf{N}_\mathbf{q}^{2D}, \mathbf{u}_\mathbf{q}^{2D}$

b. The spatial distances and the modified zenith angles used for height differences observations Δh_{ij}^{b}, forming 1D network (trigonometric leveling) → $\mathbf{N}_\mathbf{q}^{1D}, \mathbf{u}_\mathbf{q}^{1D}$ (uncorrelated with 2D NEQ)

Steps (continued):

5. NEQ for the classical network→ $\mathbf{N}_\mathbf{q}^{2D}, \mathbf{N}_\mathbf{q}^{1D}, \mathbf{u}_\mathbf{q}^{2D}, \mathbf{u}_\mathbf{q}^{1D}$

6. Same as the 5th of Scenario 1

7. Same as the 6th of Scenario 1

Height difference

$\Delta h_{ij} = \rho_{ij} \cos z_{ij} + s_i - t_i$ *s* and *t* instrument's and target's heights, respectively

Observations Equations

$$
\begin{array}{|l|}
\hline
\Delta h_{ij}^{b} - \Delta h_{ij}^{o} = h_j - h_i + v_{ij} \\
\vdots \\
\Delta h_{nm}^{b} - \Delta h_{nm}^{o} = h_n - h_m + v_{nm} \\
\hline
\end{array}
$$

$$\Rightarrow \mathbf{b} = \mathbf{Adh} + \mathbf{v} \xrightarrow{\mathbf{dh}=\mathbf{dU}}$$

$$\mathbf{b} = \mathbf{AdU} + \mathbf{v} \rightarrow$$

$$\mathbf{N}_{\mathbf{q}}^{\mathbf{1D}}, \mathbf{u}_{\mathbf{q}}^{\mathbf{2D}}$$

Scenario 3: Pure 2D network

In this scenario, we just avoid to use any 3D information, just combine the horizontal part of a GNSS network and the classical 2D network.

Steps (continued)

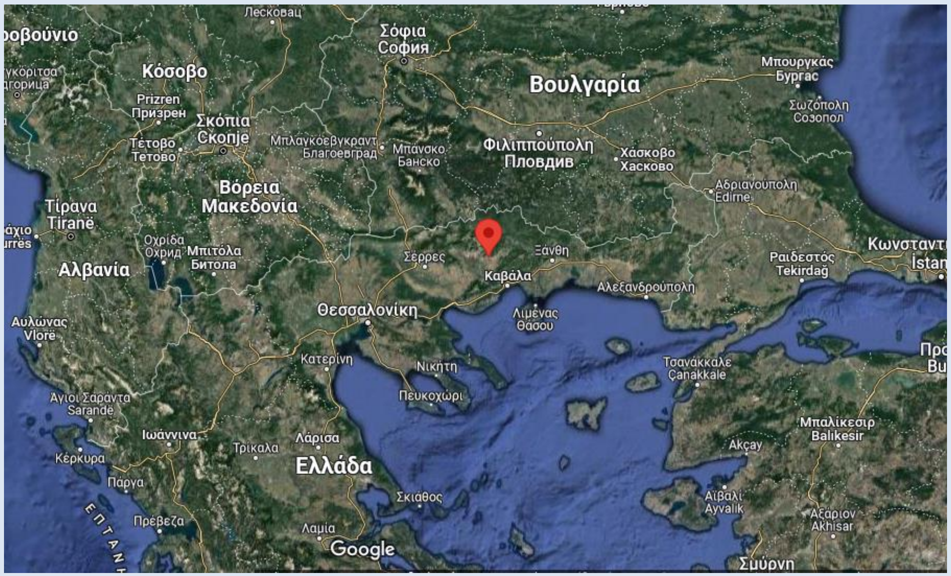
5. NEQ for the classical network→ $\mathbf{N}_{\mathbf{q}}^{\mathbf{2D}}, \mathbf{u}_{\mathbf{q}}^{\mathbf{2D}}$

6. Same as Scenario 1, but in 2D

Scenario 3 leads to a combined pure 2D network with the advantage of referring the 2D network to a modern TRF (expressed in the horizontal part of a topocentric system).

Then, the height information could obtained through spirit or trigonometric leveling (knowledge of the DoV).

Case Study : Drama, Eastern Macedonia, Greece



Responsible agency:

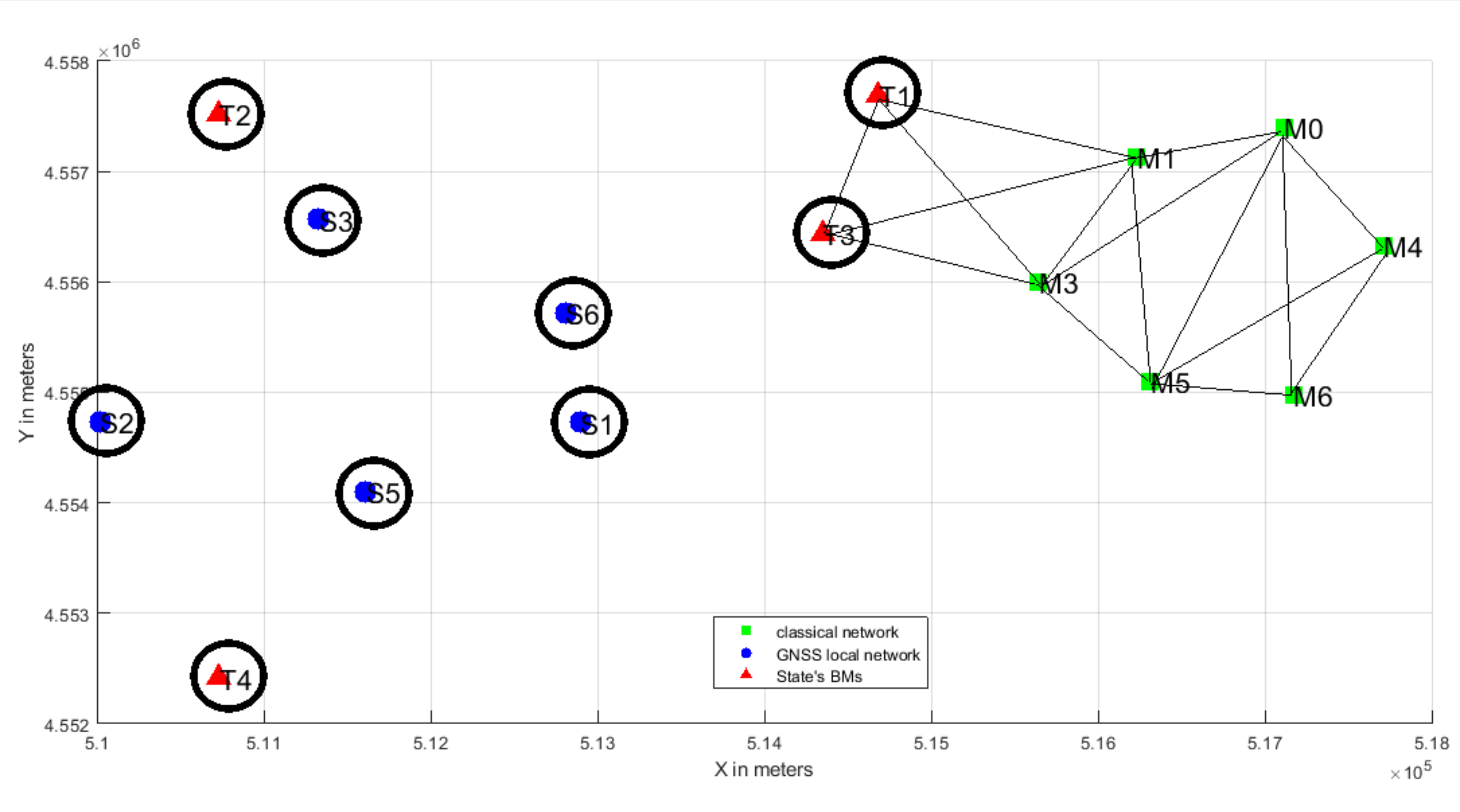
Drama's Municipal Enterprise of Water Supply and Sewerage (DEYAD)

a. classical network (1998): spatial distances, directions, zenith angles, 2 BMs of state's geodetic network occupied.

b. Horizontal angles are measured were measured in 4 observational periods (accuracy: 1 mgon). The spatial distances' accuracy was considered as 0.5 cm + 5 ppm.

c. GNSS network (2014): Static observations (min. 2 h), 5 stations in the city, 4 BMs, 1 EUREF station (DUTH in Xanthi). Reference Frame: ITRF2008, epoch 2014.35, solved in BSW 5.2→ SINEX file.

d. DoV w.r.t. XGM2019e (Zingerle et al. 2020, d/o 2190)



- Only two common stations (T1, T3)
- M0, M1, M4 located in forest, no GNSS-obs. are possible
- No visibility between the stations of the classical and the GNSS networks (for spatial connection with terrestrial data)
- At least 4 majors traverses are based on the classical network.
- Pipelines’ mapping is done only with respect to the traverses.

scenario	mean spherical error* (cm)	max. spherical error** (cm)	mean horizontal error*** (cm)	max. horizontal error**** (cm)
1st	1.85	4.45 (at point M6)	1.21	2.41
2nd	**1.01**	2.84 (at point M6)	0.85	1.22 (at point M4)
3rd	-	-	0.56	1.10 (at point M4)
GNSS network (initial)	0.67	0.95 (at point S6)	0.32	0.57 (at point S6)
2D classical network (initial)	-	-	1.08	1.74 (at point M4)

$$* \quad \sqrt{\frac{trace\left(\hat{\mathbf{C}}_{\mathbf{q}}^{\mathbf{3D}}\right)}{3n}}$$

$$*** \quad \sqrt{\frac{trace\left(\hat{\mathbf{C}}_{\mathbf{q}}^{\mathbf{2D}}\right)}{2n}}$$

$$** \quad \max\left(\sqrt{\hat{\sigma}_{E_i}^2 + \hat{\sigma}_{N_i}^2 + \hat{\sigma}_{U_i}^2}\right)$$

$$**** \quad \max\left(\sqrt{\hat{\sigma}_{E_i}^2 + \hat{\sigma}_{N_i}^2}\right)$$

Conclusions

- The 2nd scenario (=the height-related NEQ are separately estimated), gives the better results than the first one (mitigation of zenith angles uncertainty)

- The full 3D transition (1st scenario) is worse; The effect of the low accuracy zenith angles is significant regarding the accuracy

- The combination scenarios rely on the knowledge of the gravity field (DoV). Neglecting the DoVs worsen the results (severely)

- Though the accuracy is slightly worse after the combination, the results are still good enough for surveying applications and the existing infrastructure could directly refer to the combined solution→ **the whole infrastructure can be aligned to ITRF2008!**

General Remarks

- The proposed strategy can be applied in various combination scenarios between 3D and 2D networks
- It is one good choice for problems with problematic geometry of the common points.
- The combination scenarios of the proposed strategy has an advantage in the case of using SINEX files
- It can be easily applied for the "good geometry" cases
- It can be applied for the combination of any space technique (VLBI, SLR, DORIS) with terrestrial data.

Thanks for the attention

(and the patience…)

For any questions, remarks and more infos: dampatzi@teicm.gr