IAG Commission 1 International Symposium

"Reference Frames for Applications in Geosciences"

October 17-20, 2022

# *Evaluation of common-mode errors in global multi-year frames: a case study in the ITRF solution series*

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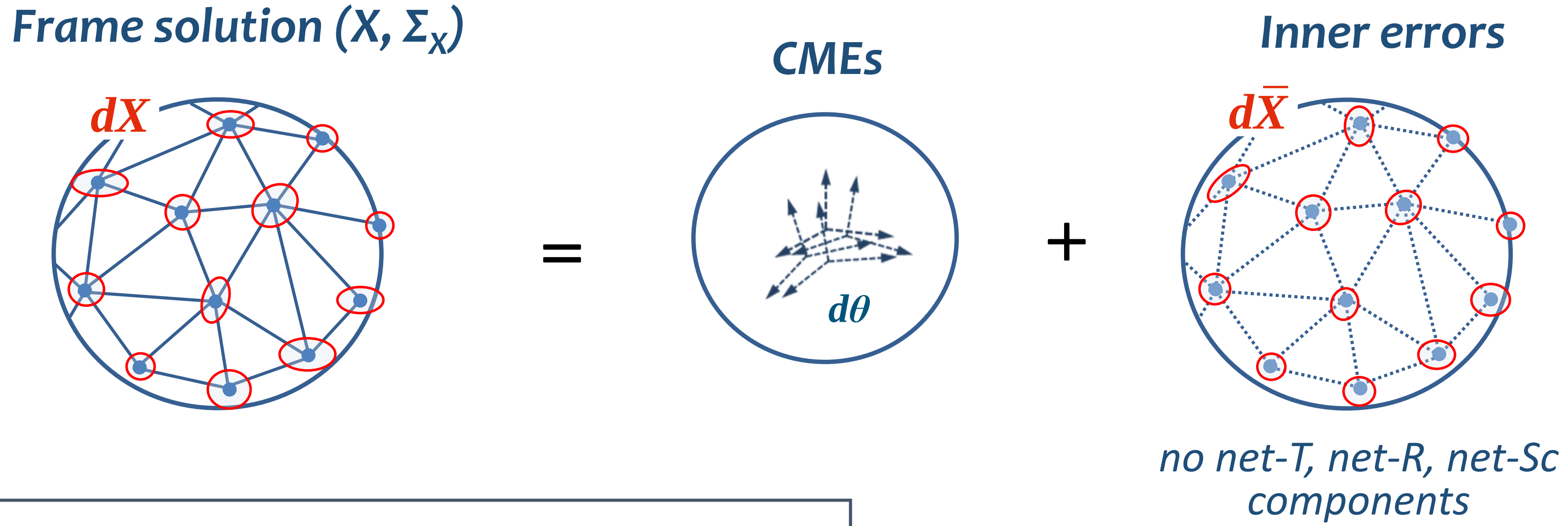
# Motivation – what we do in this study

*Given the error CV matrix for a set of estimated coordinates, **find the accuracy** of the origin, orientation & scale **of the coordinate system that is implicitly realized by these coordinates..***

## Why ?

- To infer the internal accuracy of frame solutions at coordinate system level
- To evaluate the intrinsic common-mode errors (CMEs) of frame solutions

# Motivation – what we do in this study



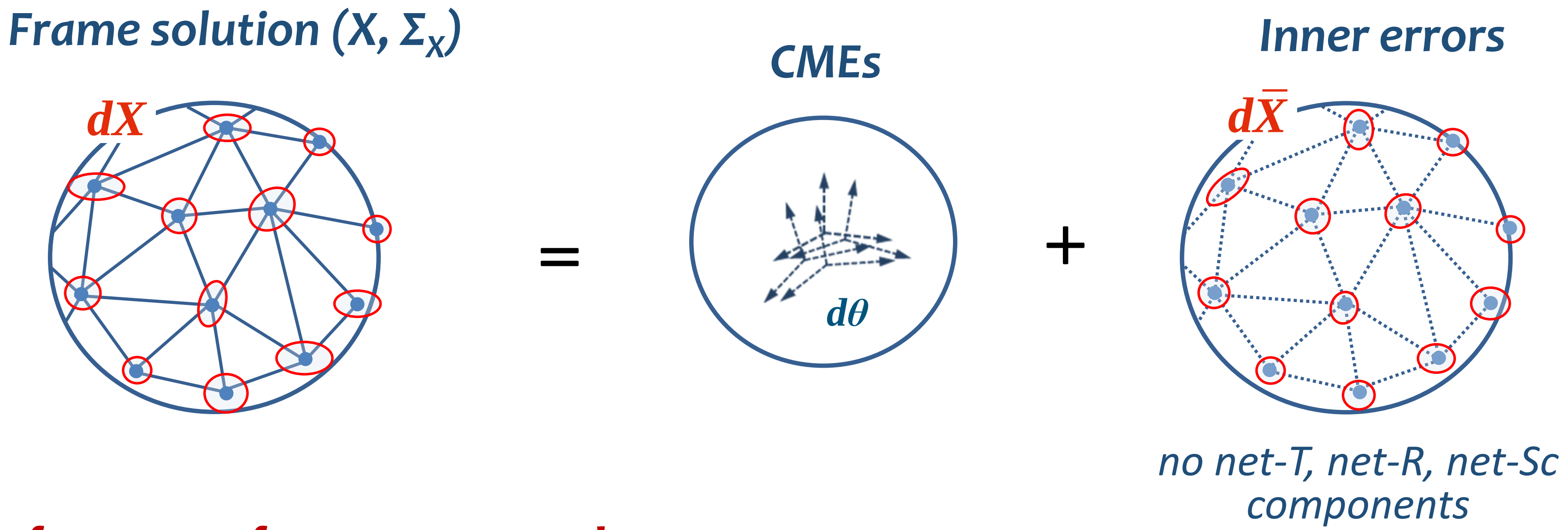

**Frame coordinate error projection:**

$$\boldsymbol{dX} = \underbrace{\boldsymbol{Q}\,\boldsymbol{dX}}_{\boldsymbol{A}\,\boldsymbol{d\theta}} + \underbrace{(\boldsymbol{I}-\boldsymbol{Q})\,\boldsymbol{dX}}_{\boldsymbol{d\bar{X}}}$$

projection matrix* $\boldsymbol{Q} = \boldsymbol{A}(\boldsymbol{A}^T\boldsymbol{A})^{-1}\boldsymbol{A}^T$

$\boldsymbol{A}$ is the Jacobian of HT (7p or 14p)

# Motivation – what we do in this study




**Inference of common-mode errors:**

$$\boxed{d\theta} = (A^T A)^{-1} A^T \boxed{dX}$$

$$\boxed{\Sigma_{d\theta}} = (A^T A)^{-1} A^T \boxed{\Sigma_X} A (A^T A)^{-1} \rightarrow$$

Assessment of (internal) frame accuracy at coordinate system level

# Case study – processing scheme

- **Input:** CV matrices of estimated station positions/velocities from ITRF SINEX solution files (releases: 2000, 2005, 2008, 2014, 2020)
- Removal of stations with bad accuracy (> 5 cm, > 5 cm/yr)
- Project CV matrices to coordinate system level – separately for each technique subnetwork
- **Output 1:** internal accuracy metrics of the realized origin, orientation and scale (and their temporal evolution) in each ITRF release
- **Output 2:** forward in-time prediction of ITRF internal accuracy

# +++ Authors' after note +++

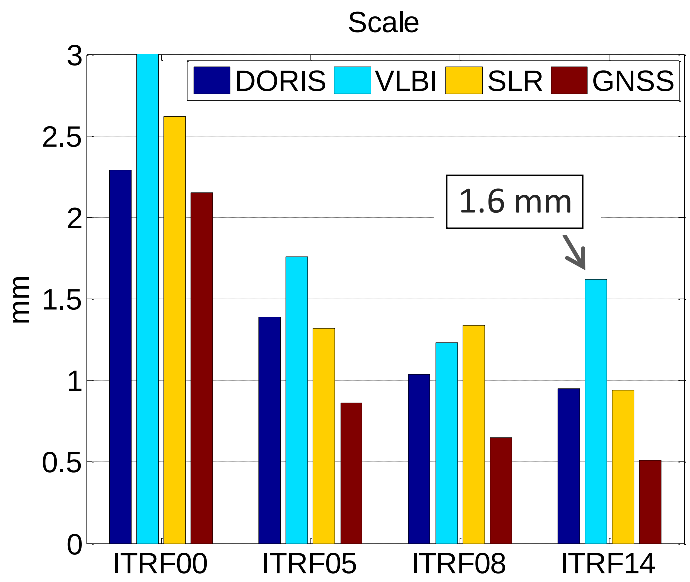
Ideally, what we would like to see in the following results is:

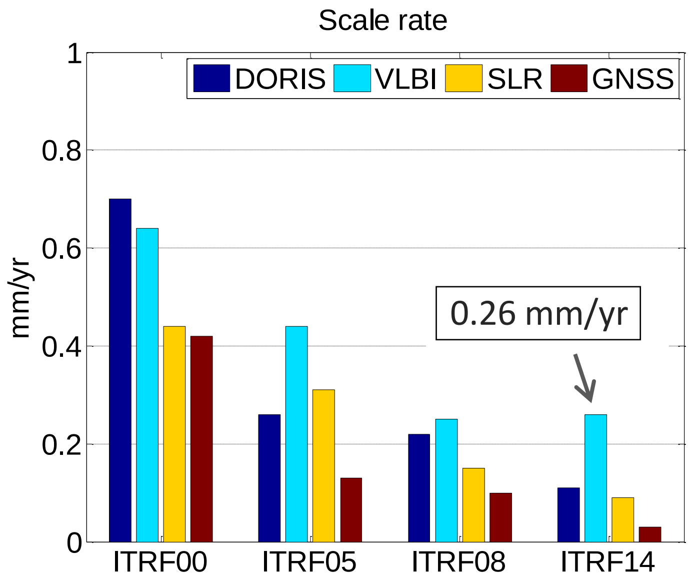
- A gradual improvement of the internal frame accuracy from one ITRF solution to the next..

- A balanced distribution of the internal frame accuracy in each ITRF solution among the different technique subnets..

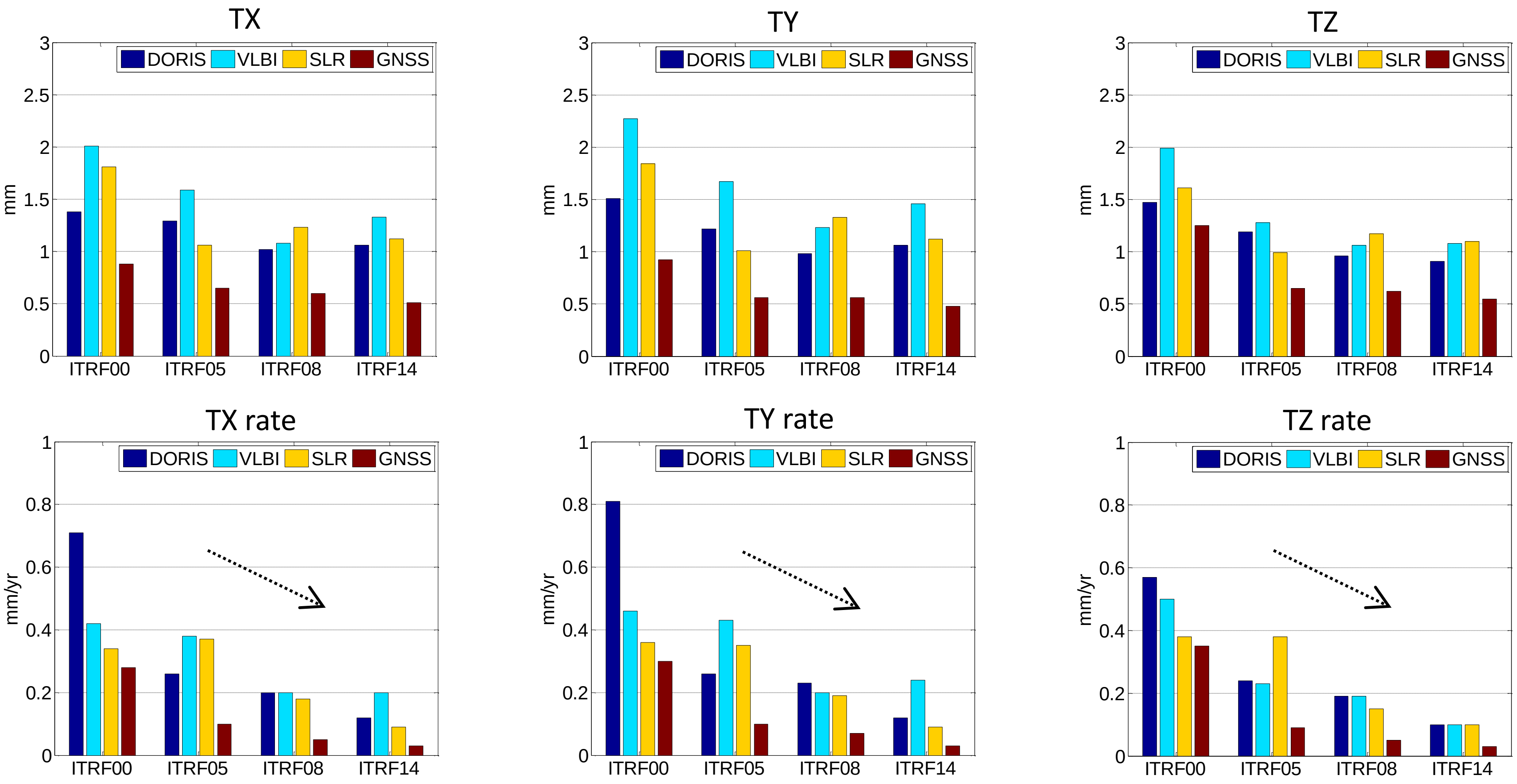
- A tendency to approach (or even exceed) the "accuracy barrier" of 1 mm and 0.1 mm/yr for the realized origin, orientation and scale in the ITRF solutions.. (GGOS specs)

# Internal accuracy in ITRF solution series

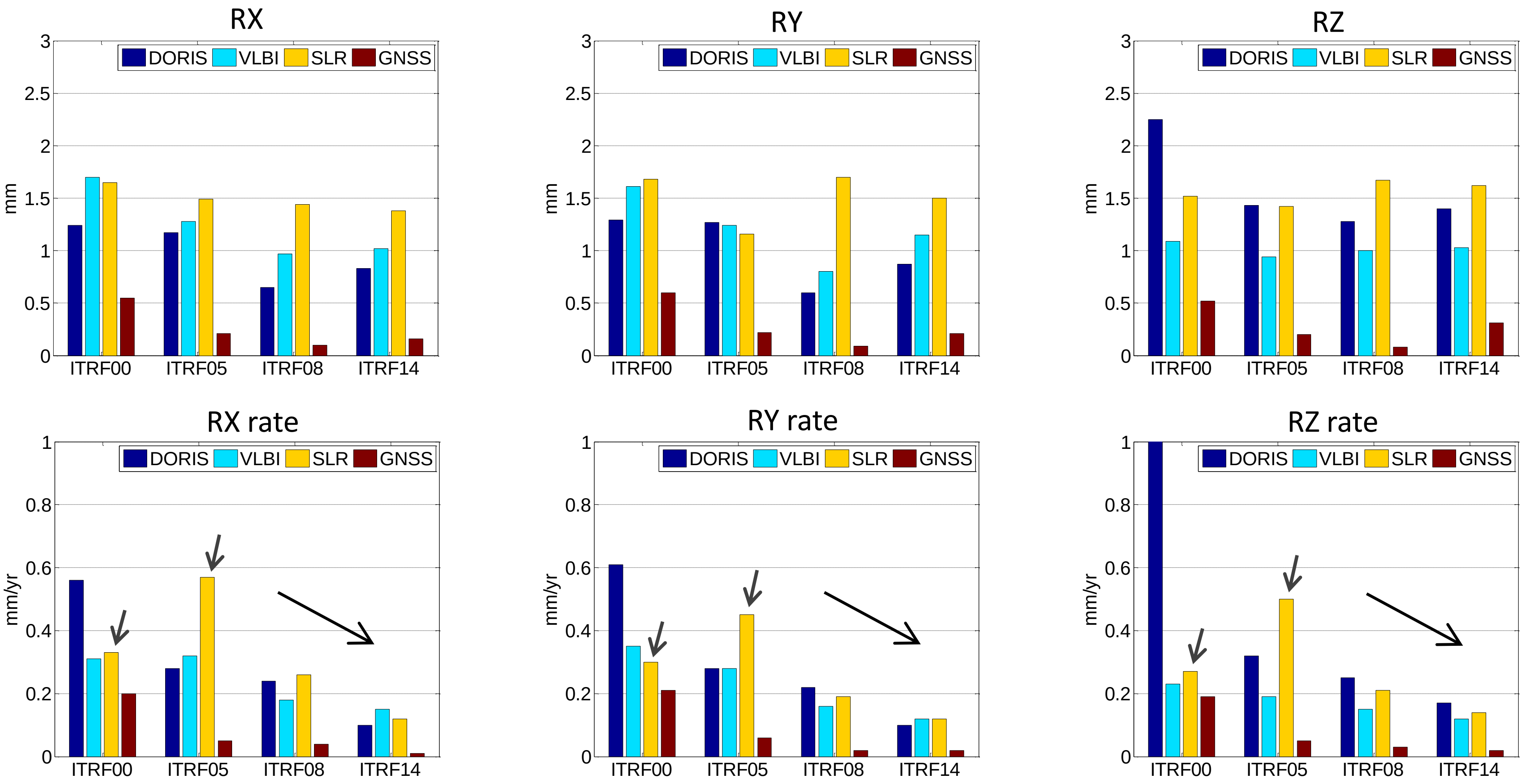


# Internal accuracy in ITRF solution series



# Internal accuracy in ITRF solution series



# Forward prediction of frame accuracy

The previous results relied on the CV mapping:

$$\Sigma_X = \begin{bmatrix} \Sigma_{X(t_o)} & \Sigma_{X(t_o),\dot{X}} \\ \Sigma_{\dot{X},X(t_o)} & \Sigma_{\dot{X}} \end{bmatrix} \longrightarrow \Sigma_{d\theta} = \begin{bmatrix} \boxed{\Sigma_{\theta(t_o)}} & \Sigma_{\theta(t_o),\dot{\theta}} \\ \Sigma_{\dot{\theta},\theta(t_o)} & \boxed{\Sigma_{\dot{\theta}}} \end{bmatrix}$$

$$\Sigma_{d\theta} = (A^T A)^{-1} A^T \Sigma_X A (A^T A)^{-1}$$

To see how the frame accuracy at coordinate system level behaves in time, we may use the following projection:

$$\boxed{\Sigma_{\theta(t_o+\delta t)}} = \Sigma_{\theta(t_o)} + \delta t^2 \Sigma_{\dot{\theta}} + \delta t \left( \Sigma_{\theta(t_o),\dot{\theta}} + \Sigma^T_{\theta(t_o),\dot{\theta}} \right)$$

The square roots of the diagonal elements of the above matrix are shown next..

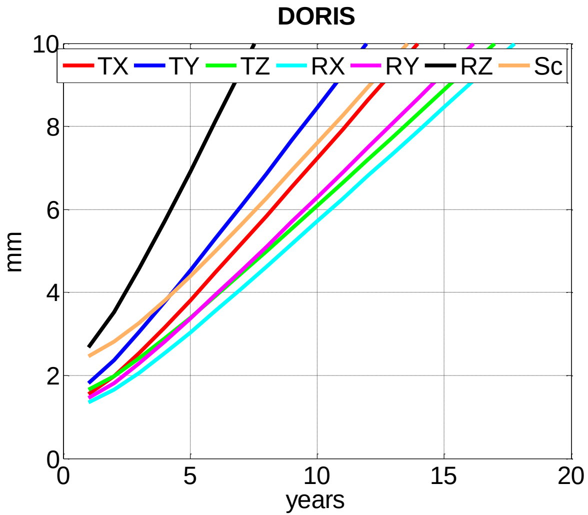
# +++ Authors' after note +++

To help you focus on the following results, keep in mind:

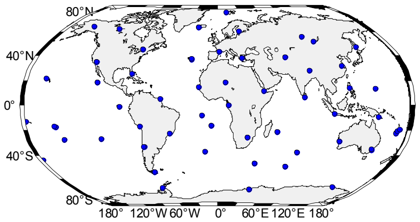
- The "zero point" in the horizontal time axis refers to the reference epoch that is associated with each ITRF solution
  ITRF2000 (to = 1997.0), ITRF2008 (to = 2005.0), ITRF2014 (to = 2010.0)

- According to the GGOS specs, the desirable frame accuracy **over a ten-year period after the reference epoch** should be ~2 mm (for all seven frame parameters). This is fully achieved in the case of the GNSS subnet (ITRF2008, ITRF2014) and partially achieved in the other 3 technique subnets (ITRF2014).

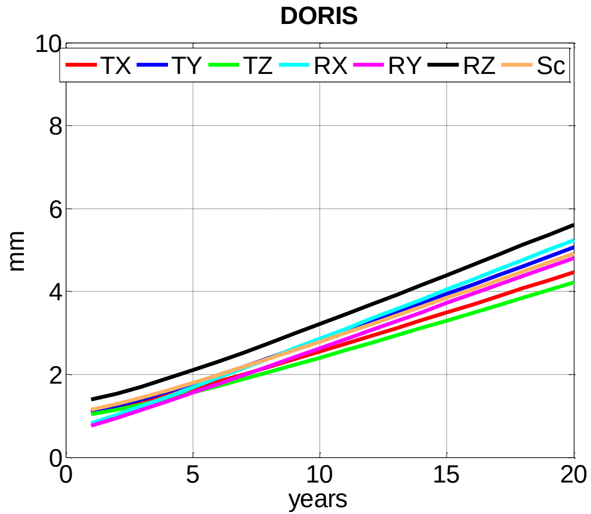
# ITRF internal accuracy evolution – DORIS



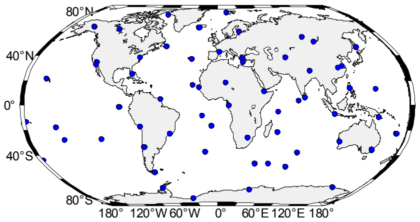

**ITRF2000**



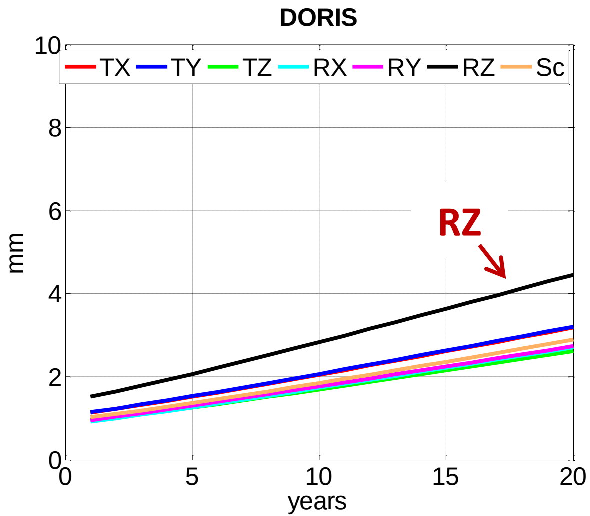

78 stations



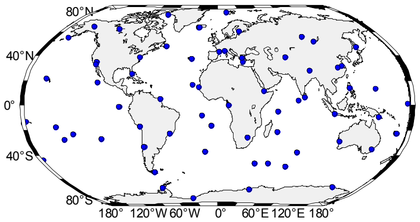

**ITRF2008**




130 stations

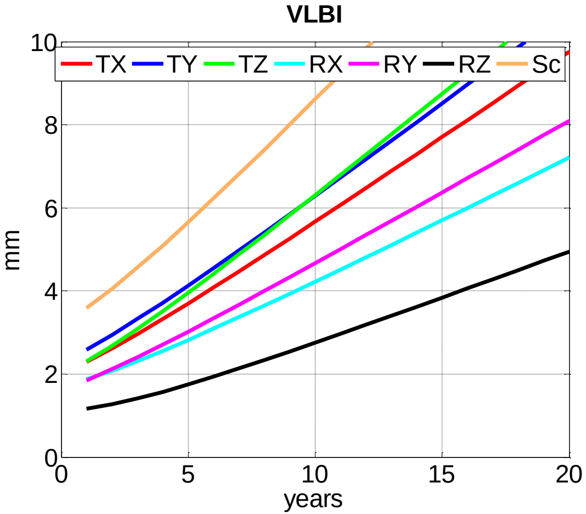



**ITRF2014**

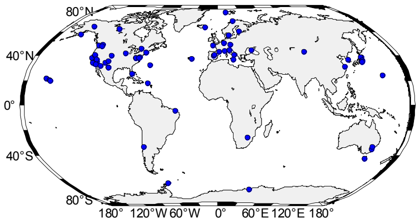



155 stations

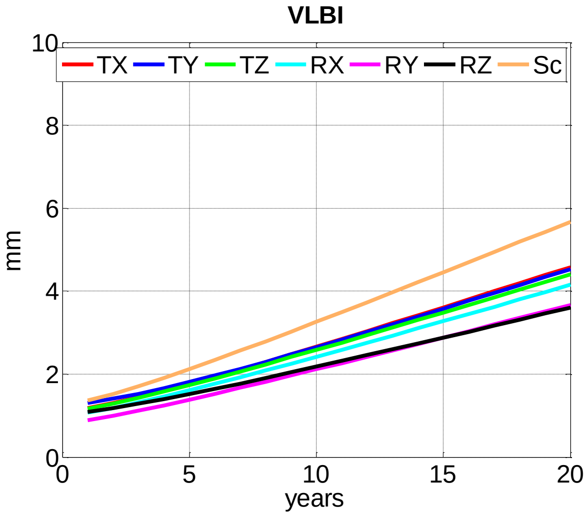
# ITRF internal accuracy evolution – VLBI



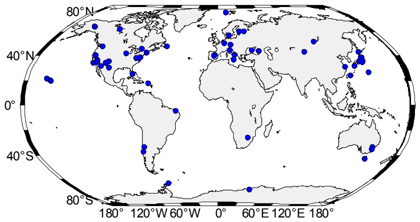

**ITRF2000**



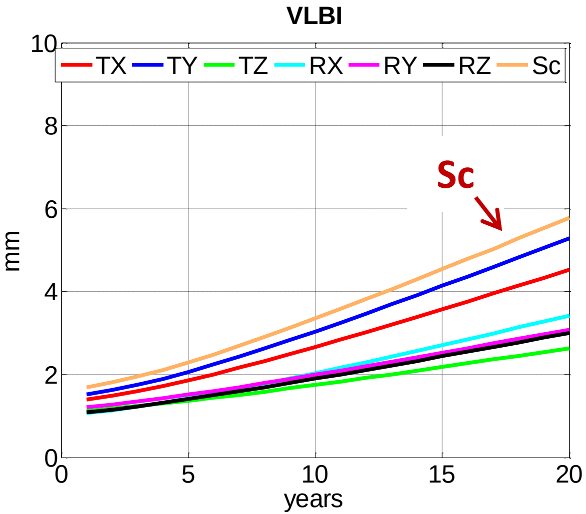
93 stations



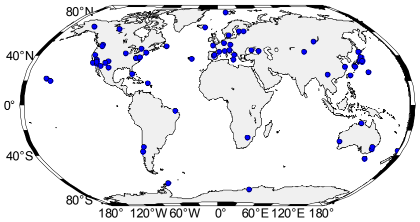

**ITRF2008**



82 stations

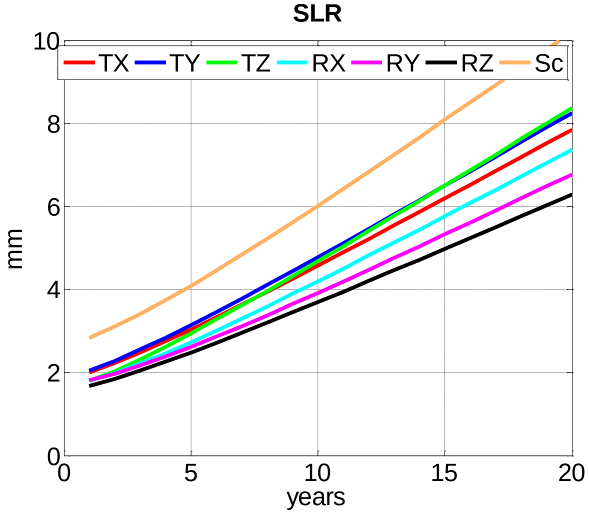



**ITRF2014**

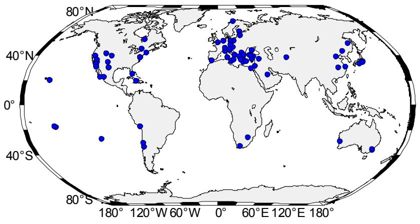


98 stations

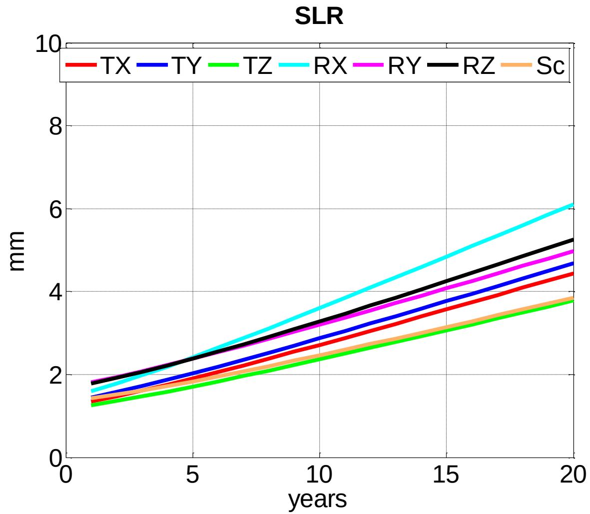
# ITRF internal accuracy evolution – SLR



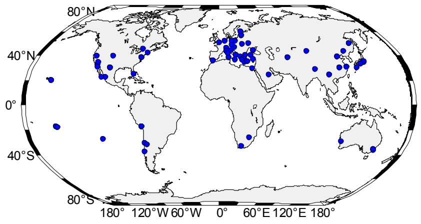

## ITRF2000



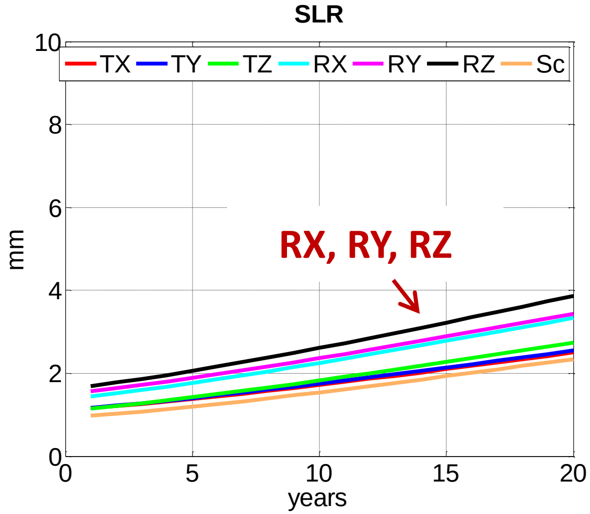

122 stations



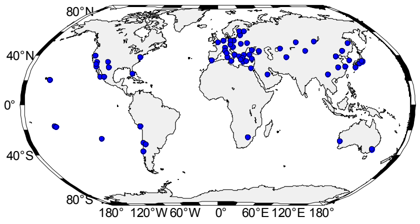

## ITRF2008




104 stations

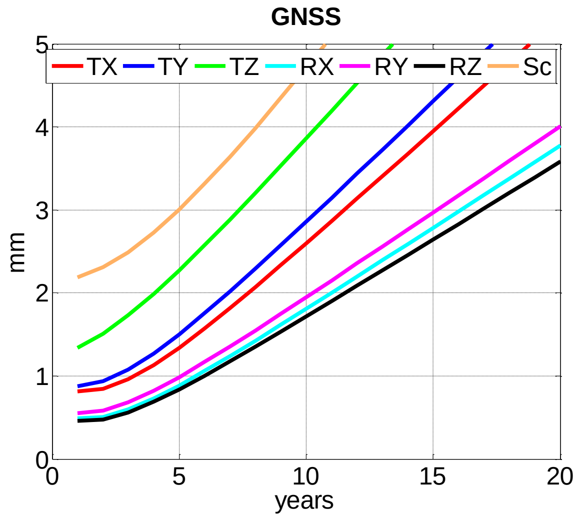



## ITRF2014

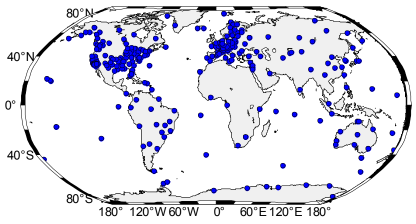



107 stations

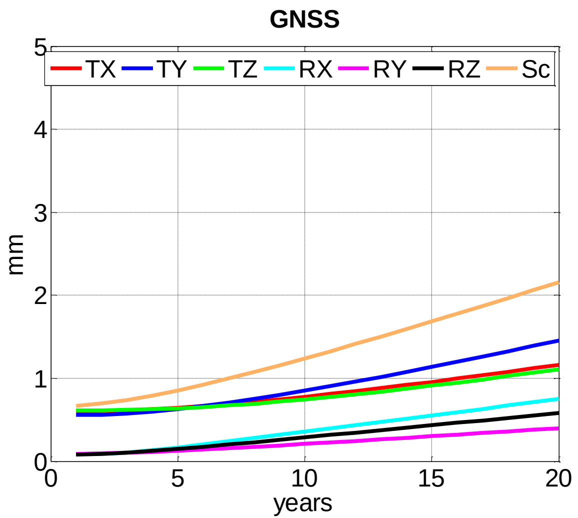
# ITRF internal accuracy evolution – GNSS



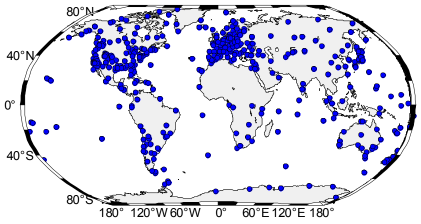

**ITRF2000**



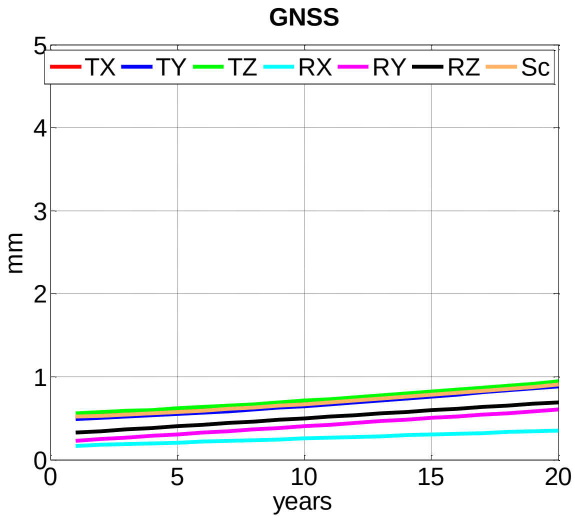
338 stations



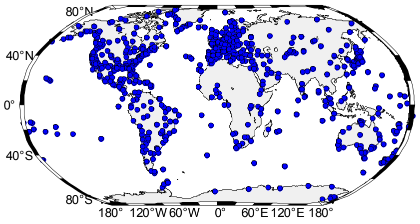

**ITRF2008**



560 stations




**ITRF2014**



1054 stations

# ITRF2020 – a truly non-secular frame

**Major innovation:** simultaneous estimation of non-secular station kinematic parameters in the final combination stage (harmonic coeffs of annual and semi-annual periodic signals)

**But,** the "over-parameterized" ITRF2020 kinematic model **may weaken the internal estimation accuracy of its secular part**!

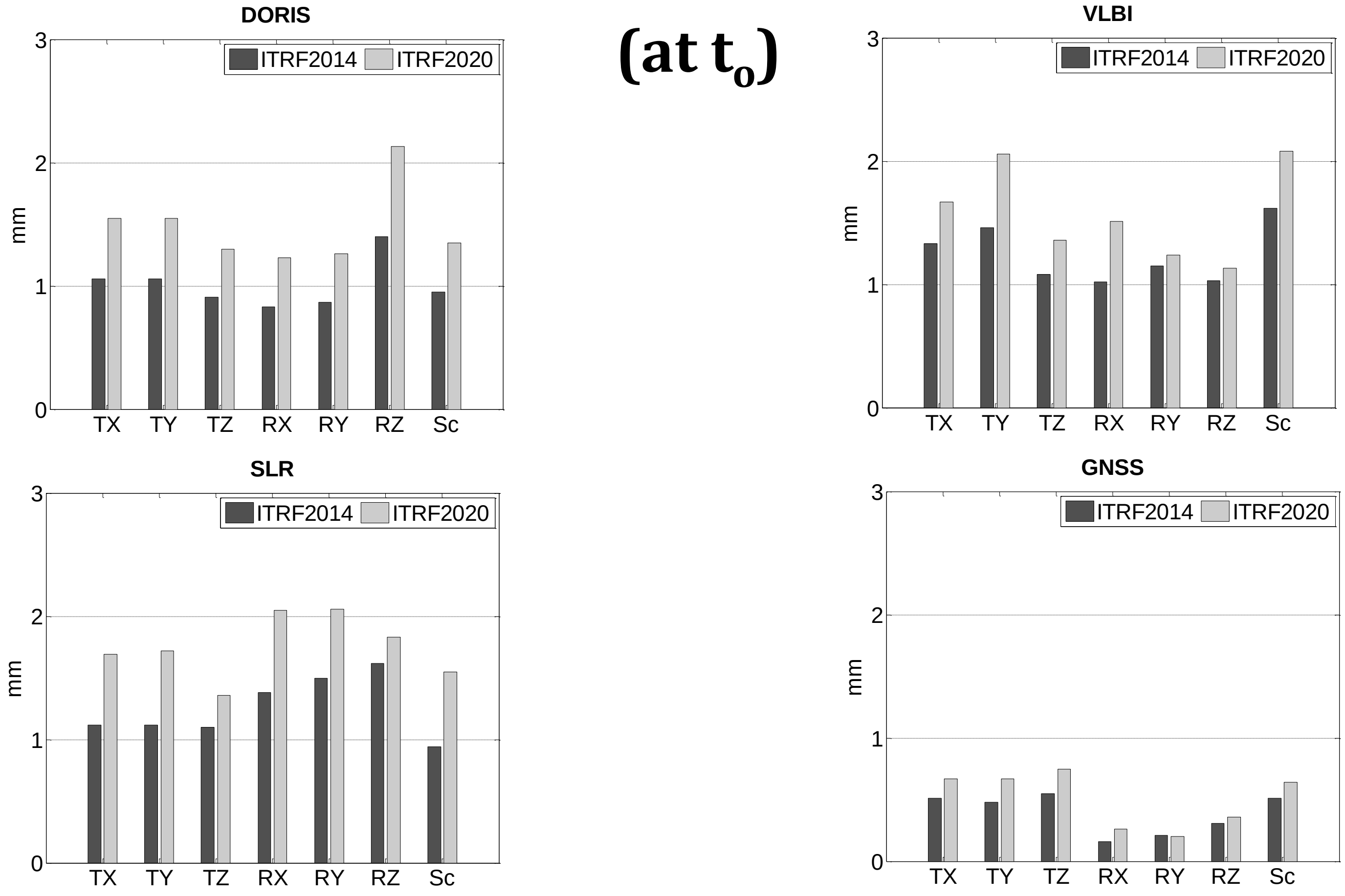
From standard LS estimation theory, we know that:

$$\boldsymbol{y} = \boldsymbol{B}\boldsymbol{x} + \boldsymbol{v} \quad \longrightarrow \quad \boldsymbol{\Sigma}_{\boldsymbol{x}} = (\boldsymbol{B}^T\boldsymbol{W}\boldsymbol{B})^{-1}$$

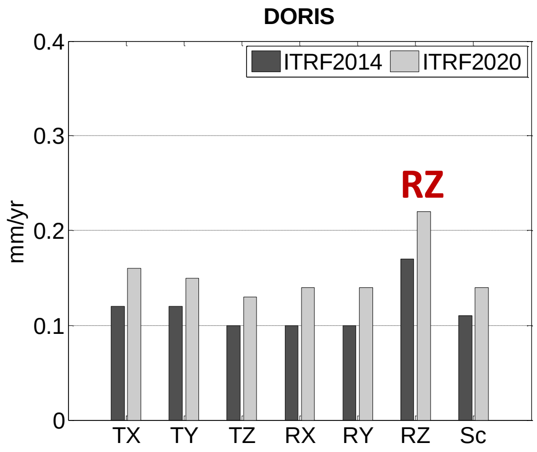
$$\boldsymbol{y} = \boldsymbol{B}\boldsymbol{x} + \boldsymbol{C}\boldsymbol{z} + \boldsymbol{v} \quad \longrightarrow \quad \boldsymbol{\Sigma}'_{\boldsymbol{x}} = \boldsymbol{\Sigma}_{\boldsymbol{x}} + \boldsymbol{\Sigma}_{\boldsymbol{x}}\boldsymbol{B}^T\boldsymbol{W}\boldsymbol{C}\underbrace{(\ldots\ldots)^{-1}}_{\text{pos. def. matrix}}\boldsymbol{C}^T\boldsymbol{W}\boldsymbol{B}\boldsymbol{\Sigma}_{\boldsymbol{x}}$$

# ITRF2014 vs. ITRF2020 – internal accuracy

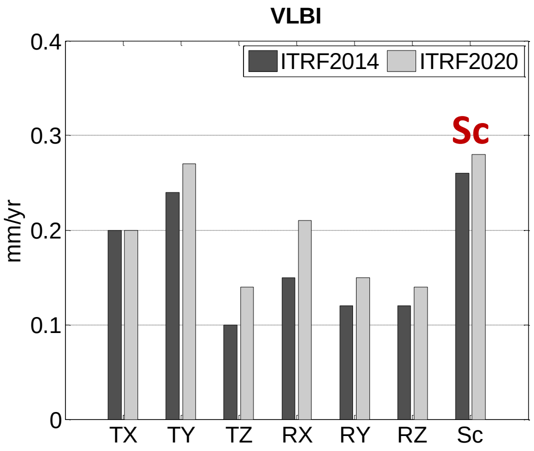
(at $t_o$)

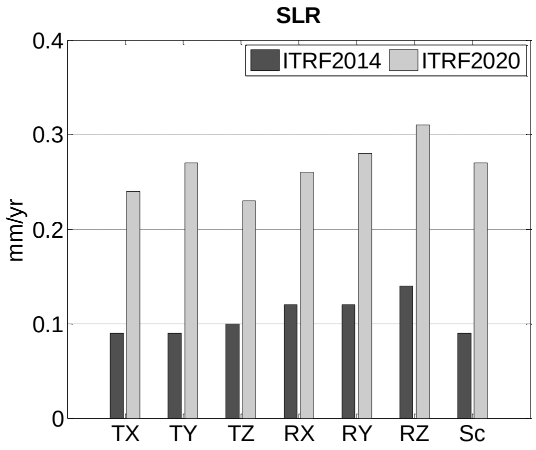


# ITRF2014 vs. ITRF2020 – internal accuracy

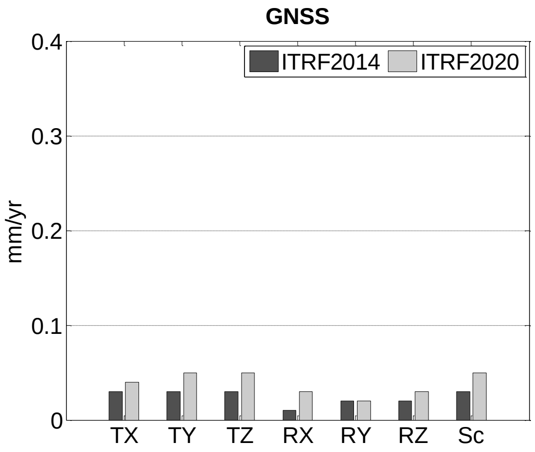



(rates)




> 50% degradation



# Summary – Conclusions

- Frame CRD estimation errors = CMEs + inner errors

  - useful error decomposition in any frame solution

  - CMEs reveal (internal) frame accuracy at coordinate system level

- GGOS specs of global TRF accuracy: 1 mm, 0.1 mm/yr (in all FPs)

  not yet achieved in ITRF – except for GNSS

- Ten-year prediction of ITRF2020 internal accuracy:

  2-4 mm for origin, orientation, scale – except for GNSS (≈ 1 mm)

  *Future work: assess internal accuracy of non-secular part of ITRF2020*